

Feasibility of Vibration-Based Long-Term Bridge Monitoring Using the I-35W St. Anthony Falls Bridge

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16. Abstract (Limit: 250 words) Vibration based structural health monitoring has become more common in recent years as the required data acquisition and analysis systems become more affordable to deploy. It has been proposed that by monitoring changes in the dynamic signature of a structure, primarily the natural frequency, one can detect damage. This approach to damage detection is made difficult by the fact that environmental factors, such as temperature, have been shown to cause variation in the dynamic signature in a structure, effectively masking those changes due to damage. For future vibration based structural health monitoring systems to be effective, the relationship between environmental factors and natural frequency must be understood such that variation in the dynamic signature due to environmental noise can be removed. A monitoring system on the I-35W St. Anthony Falls Bridge, which crosses the Mississippi River in Minneapolis, MN, has been collecting vibration and temperature data since the structures opening in 2008. This provides a uniquely large data set, in a climate that sees extreme variation in temperature, to test the relationship between the dynamic signature of a concrete structure and temperature. A system identification routine utilizing NExT-ERA/DC is proposed to effectively analyze this large data set, and the relationship between structural temperature and natural frequency is investigated.			
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FEASIBILITY OF VIBRATION-BASED LONG-TERM BRIDGE MONITORING USING THE I-35W ST. ANTHONY FALLS BRIDGE

FINAL REPORT

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EXECUTIVE SUMMARY

The new I-35W St. Anthony Falls Bridge, which opened to traffic on September 19, 2008, consists of two separate structures: parallel northbound and southbound crossings. Both structures are four-span post-tensioned concrete box girder bridges, with three continuous spans and one independent span. The southbound structure is 1227'-8⁷/₈" long and the northbound structure 1220'-3¹/₄" long. Over 500 sensors were deployed on the bridge, including vibrating wire strain gages, thermistors, linear potentiometers, accelerometers, and others. Of particular interest is the previously un-analyzed vibration data collected via the 26 accelerometers deployed on the bridge.

Accelerometer measurements have been collected at a sample rate of 100Hz since the opening of the bridge. The resulting nearly 1.5 terabytes of vibration data from the I-35W St. Anthony Falls Bridge offers a unique opportunity to investigate environmental effects on modal parameters of the concrete bridge using output-only system identification techniques. Not only does the dataset span many years, but the location of the bridge, Minneapolis, Minnesota, is one of weather extremes.

A unique challenge associated with the quality and quantity of vibration data was the detection of information-rich sections of data for system identification. Investigating the effectiveness of system identification as a function of various signal quality parameters revealed that the ratio of peak signal amplitude to signal Root-Mean-Square value (RMS) was a strong indicator of a good signal. A large peak-to-RMS ratio indicated a large excitation event followed by free vibration, whereas peak and RMS as signal quality indicators alone were unable to differentiate between instances of ambient and forced vibration with adequate energy for system identification.

In addition to signal quality identification, an algorithm for identifying and sorting the natural frequencies and mode shapes was required. A mode shape sorting technique was developed and combined with the output-only NExT-ERA/DC system identification technique to achieve this objective. NExT-ERA/DC allows for significant "tuning" by the user to optimize both the success rate at which it identifies the target modal properties of a structure and the confidence level in those results. To ensure NExT-ERA/DC was utilized efficiently, the following parameters were investigated: modal order specification, number of data points, windowing, and reference channel selection.

Using NExT-ERA/DC and the signal detection routine developed, 29,333 unique data segments were analyzed with success rates of 81.95%, 89.94%, 63.45%, and 38.88% for identifying the first four vibration modes respectively. The first three modes were found to be bending modes, and the fourth mode was found to be a torsional mode. From this large set of results, it was determined that generally as temperature increases, the natural frequency of the first, second, and fourth modes decreases, while the third mode natural frequency seems independent of temperature. All four modes appeared independent of both temporal and spatial temperature gradients. Additionally, assuming all vibration data was recorded while the bridge was in an undamaged state, the magnitude of change in natural frequency as temperature varies is too great to attribute to change in the modulus of elasticity with temperature alone. Other factors, such as humidity, which are correlated with temperature, must also be contributing to the variation in natural frequency.

The ability to extract natural frequencies and mode shapes from the vibration data of the I-35W St. Anthony Falls Bridge demonstrates the viability of using accelerometer measurements for long term monitoring of the structure. Additionally, it demonstrates that the fidelity of the measurements is sufficient to use the data with more sophisticated signal processing techniques, which may allow for estimation of bridge behavior such as vertical deflection.

CHAPTER 1: INTRODUCTION

Vibration-based structural health monitoring of bridges has garnered significant research attention due to its simple premise: changes in the dynamic signature of a structure can be correlated to damage (Adams et al. 1978; Doebling et al. 1998). The dynamic signature, including natural frequencies, mode shapes, and damping, can capture global behavior using accelerometer measurements, in contrast to the local behavior captured by strain measurements. Additionally, the complexity of structures for which modal parameters can be determined has advanced through the use of system identification techniques (Peterson 1995). As such, vibration methods have been implemented as anomaly detection on numerous structures (Webb et al. 2015). However, the viability of vibration-based monitoring has been limited due to the challenge of extracting meaningful information from the ‘noise’ in deployments (Brownjohn et al. 2011).

The dynamic signature is sensitive to measurement noise, environmental effects, modeling uncertainty and varying excitation, which often masks or mimics the modal behavior changes due to damage (Brownjohn et al. 2011; Zhang et al. 2013). For example, Cawley (1997) demonstrated that a crack through 2% of the thickness of a cantilever beam produces the same magnitude change in the first natural frequency as a 0.05% change in length. Such a change in length could be due to a change in temperature or unexpected slip at the bearings, and demonstrates the difficulty associated with system identification as a means for damage detection. An early field illustration of this variability in natural frequency with temperature was the monitoring of the Alamosa Canyon Bridge, a steel stringer bridge in southern New Mexico (Cornwell et al. 1999). Vibration data collected over two 24-hour test periods showed that the first natural frequency varied by approximately 5% during the first test and the natural frequencies of the first three modes varied by 4.7%, 6.6%, and 5.0% respectively, in the second. Given the significant variation, compensation for these unknowns in the dynamic signature is essential for adoption of dynamic signatures for infrastructure monitoring.

Limited research in vibration-based monitoring has tried to both capture the environmental effects and establish their cause for future compensation. Xia et al. (2006) identified the mode shape, natural frequency, and damping of a simply supported single span reinforced concrete slab under varying temperature and humidity conditions. They found that natural frequency decreased and damping increased as temperature increased, however the standard deviations of damping results were too large to easily quantify the relationship between damping and temperature. They also found that identified mode shapes did not appear to vary with temperature. Xia et al (2012) hypothesized that this correlation between temperature and natural frequency is due to variation in the concrete modulus of elasticity. The monotonic relationship between temperature and natural frequency was also visible in the year-long monitoring of the Z24-Bridge, a post-tensioned concrete box girder bridge in Switzerland (Peeters and De Roeck 2001). However, with the longer monitoring period, they observed a distinctly bi-linear relationship between temperature and natural frequency of the first two modes (2001). As temperature increased from below freezing to 0°C, the natural frequency decreased linearly. From 0°C and up, the natural frequency still decreased linearly with temperature, but at a reduced rate. They

concluded that the asphalt overlay only contributed significant stiffness to the system during periods of cold temperature, thus the observed bilinear behavior.

The vibration data from the I-35W St. Anthony Falls Bridge, located in Minneapolis, MN, presented a unique opportunity to evaluate the environmental effects on a new concrete bridge structure given the seven year long duration of monitoring and location in a region with weather extremes. The challenges with practical implementations, including quality and quantity of vibration data, were addressed with the development of a system-identification process to determine information rich sections of data. The system identification results for five years of data on the undamaged structure are used to investigate the temperature-based response of the structure and other phenomenon that might be responsible for large variations in the natural frequency under changes in temperature.

CHAPTER 2: BRIDGE DESCRIPTION AND INSTRUMENTATION

The new I-35W St. Anthony Falls Bridge, which opened to traffic on September 19, 2008, consists of two separate structures: parallel northbound and southbound crossings. Both structures consist of four spans of post-tensioned concrete box girders as shown in Figure 1. Spans 1 through 3 are continuous, separated from span 4 by an expansion joint. Spans 1, 3, and 4 were cast-in-place, whereas the river span, span 2, was constructed using segmental precast construction. Each bridge is 90ft-4in wide and carries 5 lanes of traffic. Over 500 sensors were deployed on the bridge, including vibrating wire strain gages, thermistors, linear potentiometers, accelerometers, and others. The monitoring system has been collecting data since the opening of the bridge in 2008. This large dataset represents a unique opportunity to investigate bridge behavior over multiple years, seasons, and conditions, with the underlying objective of developing structural health monitoring protocols using long-term data from an in-service structure. In addition to the monitoring system, a detailed three dimensional finite element model of the southbound structure was built (French et al. 2012). The basic geometry of the model as well as the predicted mode shapes of the first four modes is presented in Figure 2. Modes 1 through 3 are bending modes and mode 4 is a torsional mode. The theoretical natural frequencies of the first four modes, using finite element analysis, were found to be 0.750 Hz, 1.464 Hz, 2.143Hz, and 2.414 Hz respectively (French et al. 2012). The model was validated for static behavior only.

Of particular interest in this work is the long-term vibration behavior of the structure, which has not been investigated previously. As part of the monitoring system, 246 thermistors and 26 accelerometers were installed, particularly in the river span of the southbound structure. The center of the river span was heavily instrumented with thermistors in order to get detailed information on the temperature gradients through the cross section (Fig. 3). Hourly temperatures have been recorded as the temporal average of five readings during the first 15 minutes of every hour for each thermistor.

Twenty-six Kistler 8310B2 accelerometers were deployed on the bridge, located on both the north and southbound structures, with twenty located in the southbound structure. Twelve accelerometers were permanently positioned along the centerline of both boxes at midspan of spans 1, 2 and 3. Fourteen additional moveable accelerometers were located in the exterior box of the southbound structure. The moveable accelerometers were placed in the current configuration shown in Figure 4 on May 11, 2010. All accelerometers were oriented vertically except for Acc 8 and Acc 10, which were oriented transversely. Acceleration signals were passed through a low-pass filter with a cutoff frequency of 159 Hz, and then digitized to 16 bits at 1000 Hz. The digitized data was then passed through a low-pass Kaiser window filter with a cutoff frequency of 23 Hz and decimated to 100 Hz. One accelerometer (SN 20607900 failed and was not recorded after February 6, 2013. Several outages of the southbound accelerometers have occurred, most recently from 7/5/2015 to 7/28/2015. A complete list of outages, including the northbound accelerometers, can be seen in Appendix A. The accelerometers have a range of ± 2 g, a frequency response of zero to 250 Hz ($\pm 5\%$), and a noise density of 38 $\mu\text{g}/\sqrt{\text{Hz}}$. DC sensors as opposed to AC sensors were chosen with the intention of measuring static deflection of the structure, however, the typical ambient vibration signal amplitude of 10 mg, which lies on the 1g DC signal, uses a

small portion of the analog-to-digital conversion range. The resulting low signal-to-noise ratio makes this integration difficult (French et al. 2012).

CHAPTER 3: SIGNAL PROCESSING

One of the objectives of the current work was to develop an algorithm to identify segments of acceleration data that could be used to consistently find the first four natural frequencies and mode shapes with high confidence. Given the quantity and quality of the vibration data, the algorithm for identifying natural frequencies and mode shapes in an efficient and effective manner was not straightforward. A mode shape sorting technique, confidence measure, and process to identify a good signal were combined with the output-only NExT-ERA/DC system identification technique to achieve this objective.

NExT-ERA/DC allows for significant “tuning” by the user to optimize both the success rate at which it identifies the target modal properties of a structure, and the confidence level in those results. To ensure NExT-ERA/DC was utilized efficiently, the following parameters were investigated: modal order specification, number of data points, windowing, and reference channel selection.

3.1 NExT-ERA/DC

NExT-ERA/DC returns modal parameters such as mode shape, damping ratio, and natural frequency, given time history of acceleration, and relies on a combination of two techniques: the Natural Excitation Technique (NExT) and the Eigen Value Realization method (ERA).

NExT allows the user to generate an impulse response function response from the ambient output response data for use with other techniques such as ERA (James III, Carne, and Lauffer 1993). NExT was implemented by taking advantage of the MATLAB built in cross power spectral density (CPSD) and inverse Fourier transform (IFFT) functions.

ERA, first developed by Juang and Pappa in the 1980’s (Juang, Cooper, and Wright 1988), allows for the extraction of modal parameters of a dynamic system through the assembly of the state transition matrix, $\mathbf{A}$, which incorporates the eigenvalues and eigenvectors of a dynamic system in state-space form (Juang, Cooper, and Wright 1988). ERA/DC is an expansion of this technique to include direct correlation, which allows for a reduction in system bias due to noise without mode overspecification (Juang, Cooper, and Wright 1988). Additionally, it has been demonstrated that ERA/DC is always at least as accurate as ERA alone (Nayeri et al. 2009). Further discussion of NExT-ERA is presented in Appendix B.

3.2 MODAL ORDER

As twenty accelerometers are present on the southbound structure, the maximum number of measureable modes using NExT-ERA/DC is twenty. As ERA/DC was utilized as opposed to ERA, the need for overspecification was reduced, and eventually it was determined unneeded. For the next ERA/DC routine implemented on the I-35W Bridge, an assumed modal order of forty was utilized. An assumed modal order of forty represents twice the number of mode shapes detectable with twenty accelerometer to account for results being returned in pairs. As higher order modes are more difficult

to identify consistently, the first four modes were specifically targeted out of all the 20 possible modes identified.

3.3 NUMBER OF DATA POINTS AND WINDOWING

Assuming the average vehicle travels at approximately 60 mph, it would take about 15 seconds to cross the I-35W St. Anthony Falls Bridge. However, the signal used needs to include free ambient vibration in addition to the traffic inciting the response. Additionally, more data points in the signal should improve the output quality as the number of averages in the CPSD increases. Therefore 3 minutes of data, or 18,000 data points recorded at 100 Hz, was found to be sufficient for system identification and balance of signal length and vibration amplitude.

The NExT routine requires cross-power spectral density, which necessitates a window type, size, and percent overlap to calculate the frequency. For a structure with the first natural period T_1 , the frequency of the first two modes identified using NExT-ERA/DC converge with a window size of $10T_1$, and the measured mode shapes converge at $25T_1$ (Nayeri et al. 2009). Assuming the first natural frequency of the southbound structure was approximately 1.33 seconds, a window size of $25T_1$ represented approximately 3325 data points, or 33 seconds. This was rounded to the next power of 2, 4096, to optimize the digital Fourier transform and allow for several averages. Various combinations of 4096 points, 8192 points, 50% overlap, 75% overlap, and 87.5% overlap were tried. Ultimately, a Hanning window of 4096 points and 75% overlap were selected for implementing NExT on the three-minute vibration signals.

3.4 REFERENCE CHANNEL SELECTION

Improper selection of reference channels can reduce the ability of NExT to separate noise from ambient vibration, therefore reducing the ability of NExT-ERA/DC to identify modal parameters. Two methods have been previously proposed: selecting a combination of reference channels that will be excited under all the target modes or selecting individual reference channels and combining NExT-ERA/DC results based on each reference channel individually (Nayeri et al. 2009). A hybrid method was implemented for the I-35W St. Anthony Falls Bridge data. Three individual reference channels as well as one combination were utilized, meaning each 3 minute data segment was analyzed using NExT-ERA/DC four times with different reference channel sets, and the results were then combined. The three individual reference channels chosen were SB SP 1 Ext, SB SP 2 Int, and Acc 6. The combination used was Acc 2, Acc 3 and Acc 4. SB SP 1 Ext was selected to target modes 2 and 3, as the sensor is located near the location of maximum relative displacement for those two modes. Acc 6 was chosen to target mode 1 and SB SP 2 Int was chosen to target mode 4, because both are located near locations of maximum relative modal displacement, respectively. The combination of Acc 2, Acc 3 and Acc 4 was selected as all three accelerometers represented locations of non-zero displacement for the four mode shapes in question. Reference channel location with respect to mode shape is shown in Figure 5. Finally, to combine results from the independent reference channels as well as the reference channel combination, the result with the highest level of confidence, as measured by the consistent mode indicator (CMI), for a particular mode from a given 3 minute segment of data analyzed was assumed to

best represent the system behavior for that mode. The results of a detailed investigation of reference channels are presented in Appendix D.

3.5 CMI

To ensure confidence in the algorithm output, the consistent mode indicator (CMI) was utilized. For an in-depth description of CMI, see Appendix C. CMI is a measure of both the temporal and spatial consistency of an identified mode shape, and its developers suggest that values above 0.8 represent high confidence results (Pappa, Elliot, and Schenk 1993). Other work has shown cutoff value closer to 0.5 can also be appropriate (Farrar et al. 1997). A CMI cutoff of 0.7 was adopted given the quality of the vibration data available. Any results returned by NExT-ERA/DC with CMI values below 0.7 were not considered.

CHAPTER 4: MODE SORTING

Given the closely spaced natural frequencies predicted by the FEA model and the comparatively small variation in mode shape as shown by Xia et al. (2006), a mode shape sorting technique was crucial to categorize the mode shapes and corresponding frequencies and damping ratios returned by the system identification technique. To sort the results by mode, the calculated mode shapes were first normalized such that accelerometer Sb SP 2 Ext had a positive relative displacement, and the largest relative displacement of any degree of freedom was set to 1.00. Utilizing the predicted mode shapes from the finite element analysis of the bridge shown in Figure 2 (French et al. 2012), assumed displacement vectors were created for each mode of interest. Assumed displacement vectors contained only 1, -1, and 0. Each degree of freedom was assigned a 1 for an assumed upwards relative displacement, a -1 for assumed downwards relative displacement, and a 0 for locations with a relative modal displacement of less than 20% of the maximum or within 30 feet of a node. Assumed displacement vectors are shown in Table 1.

For any target mode, an element wise product of the assumed displacement vector with a calculated mode shape vector returned a vector of all values greater than or equal to zero, if the calculated mode shape was that of the target mode. If any value of the product vector was less than zero, the calculated mode shape was assumed to not be the mode shape of the target mode. The frequency and damping results corresponding to an identified mode shape were then sorted accordingly.

CHAPTER 5: SIGNAL DETECTION

A contributing cause of error and uncertainty associated with output only system identification is unknown loading conditions. Multiple loading events, including but not limited to wind and traffic, can excite a structure. Due to the low profile and relatively large stiffness of the I-35W Bridge, the primary dynamic excitation event was assumed to be traffic. While traffic has the advantage of frequent excitation events as traffic routinely crosses the structure, traffic is also associated with many unknowns such as vehicle weight, position, speed, and number of vehicles crossing the structure simultaneously; all of these traffic variables are difficult to monitor and were not monitored on the I-35W St. Anthony Falls Bridge. Additionally, the dataset contains a large amount of redundant data due to its size, so an intelligent method of selecting data segments for further analysis was vital. As such, an algorithm for identifying the data segments with necessary signal to noise ratio and characteristics to be used to identify modal frequencies was implemented.

Two primary signal parameters were investigated, peak amplitude and root mean square (RMS) value. RMS was considered to be a measure of the intensity of free vibration. To investigate these parameters, accelerometer Acc 7 was selected for signal detection as it is located near midspan of the river span where several modes have high amplitude. Other accelerometer locations were considered, however accelerometers near midspan of the river span provided a better representation of the vibration response measures than any other accelerometer investigated.

Initially, peak signal amplitude alone was investigated as a means to identify good signals for analysis. The peak signal measured at Acc 7 was used to identify the best signal from 11:00 am to 1:00 pm daily from April 2010 to July 2015. One minute of data before the peak signal and two minutes after were then used to identify the modal parameters of the first four modes. Given the signals identified, RMS was determined using 100 points of data before and 900 points after the peak signal to ensure that free vibration response was captured while minimizing the noise contribution to the RMS value.

To determine what constituted a good signal for analysis, the success rates at which NExT-ERA/DC was able to identify the first four modes were calculated as a function of peak and RMS values, and are listed in Table 2. The inverse correlation between peak and success rates indicated that large peak signal alone is not sufficient to detect a good signal for analysis. Additionally, as both Peak and RMS increased, the success rates generally decreased. These results seem unintuitive, but it is possible that large RMS values are associated with periods of heavy traffic on the bridge. Heavy traffic can add mass to the structure, which could “smear” the spectral peaks as well as change the natural frequency (Steenackers and Guillaume 2005). Furthermore, it is possible that under heavy traffic, shortly spaced large vehicles continually cause forced vibration of the bridge with little-to-no free vibration. In general, free vibration, as opposed to forced vibration, is best suited for system identification using NExT-ERA/DC.

To further understand how to capture a good free vibration signal, success rates at which NExT-ERA/DC was able to identify the first four modes were calculated as a function of the ratio of peak signal amplitude to RMS value, and are listed in Table 2. As the ratio of peak to RMS increased, the success rates increased for modes 1, 2, and 4. The ratio appears to have little effect on the ability of Next-

ERA/DC to identify mode 3, which can likely be attributed to the use of reference channels which were not optimized to identify mode 3.

To illustrate this peak and RMS relationship, Figure 6 shows two 10-second vibration data segments from the I-35W St. Anthony Falls Bridge. The left plot has lower RMS than the right plot, but shows more free vibration and a larger peak-to-RMS ratio. Given that the success rate increases as the peak-to-RMS ratio increase, as well as the behavior shown in Figure 6, it appears that signals with large peak-to-RMS ratios correlate with large excitation events followed by undisturbed free vibration. Additionally, the results listed in Table 2 illustrate that NExT-ERA/DC identified frequencies and mode shapes best when the peak-to-RMS ratio was above 6. Using this information, a signal detection routine was implemented which searched time histories of data for the three minute segment, one minute before the peak and 2 after, with the greatest peak-to-RMS ratio.

CHAPTER 6: SYSTEM IDENTIFICATION RESULTS

The NExT-ERA/DC based algorithm was implemented on vibration data collected from April 2010 to July 2015 on the I-35W St. Anthony Falls Bridge. A signal detection routine was used to detect the best signal for each 60 minute block of data (eg 7:00am to 8:00am). Each three minute segment of data was analyzed using NExT-ERA/DC four times, once for each reference channel combination. The results were then sorted using the sorting technique described previously. This procedure returned results for 29,333 data segments. Mode 1 was found 82% of the time, Mode 2 was identified 90% of the time, Mode 3 was identified 63% of the time, and Mode 4 identified 39% of the time. The results are summarized in Table 3. As expected, the natural frequencies had a significantly lower coefficient of variation than the damping ratio. The theoretical natural frequencies of the first four modes, using finite element analysis, were found to be 0.750 Hz, 1.464 Hz, 2.143Hz, and 2.414 Hz respectively (French et al. 2012). All identified natural frequencies were within 9.0% of the theoretical value. Also identified were the corresponding mode shapes of the first four natural frequencies, as shown in Figures 7 through 10. The first mode was found to be the primary bending mode, and mode 4 was found to be the primary torsion mode. Modes 2 and 3 were also bending modes, with three and four inflection points respectively.

The variation between identified and theoretical natural frequencies could be reduced through model updating; the F.E. model was optimized for static superimposed loading and was not previously verified for dynamic loading. Small adjustments to the structure's dead load in the F.E. model would likely improve the dynamic results with very little impact on the static F.E. predictions. Comparing Tables 2 and 3, it appears that Mode 4 was found with a reduced success rate after the implementation of the signal detection routine. This difference is likely attributable to two components of the system identification routine at the time of the investigation into signal detection. Firstly, an appropriate CMI cutoff had yet to be determined, and therefore a CMI cutoff was not yet programmed into the system identification routine. Secondly, the sorting routine described above was not yet developed, and modes were sorted by magnitude of natural frequency as opposed to mode shape. Both factors could have contributed to fictitious modes or even mode 3, being incorrectly sorted as mode 4.

For each month from April 2010 to July 2015, the monthly average natural frequencies of the first four modes were calculated. The results, including standard deviation, are plotted in Figures 11 through 14. Gaps in the data represent southbound data collection system outages. Modes 1, 2, and 4 exhibit distinctly sinusoidal behavior, with higher natural frequencies in the winter months and lower natural frequencies in the summer months. This sinusoidal behavior suggests that there is an environmental factor, or factors, which affect the natural frequency of the structure and vary in predictable manner with the seasons. For example, temperature is usually greater in the summer months, and relative humidity is usually greater in the winter months in Minnesota. It has previously been shown that as temperature decreases, natural frequency increases (Moser and Moaveni 2011), which corresponds to the variation in the monthly average natural frequency assuming temperatures are lower in the winter months. On the contrary, mode 3 does not appear to exhibit the same sinusoidal variation in natural frequency as the other three modes.

CHAPTER 7: TEMPERATURE EFFECTS

To further understand the variation in natural frequency, the average natural frequency of each of the first four modes as a function of average structural temperature was calculated and plotted in Figures 15 through 18 respectively. Temperatures were calculated assuming a weighted average through the thickness of the deck based on cross sectional area per Eqn. (1), and rounded to the nearest whole degree Celsius. As midspan of the river span is the most heavily instrumented location, average temperature calculated at this location was assumed to be representative of the entire structure. Thermistors utilized as well as their location are presented in Table 4, where x is the transverse offset from the centerline of the box over which they are located and y is the depth with respect to the top of the deck (French et al. 2012). Thus, in Eqn. (1), T is the thermocouple readings along the midspan cross-section and A is the corresponding cross-sectional area.

$$T_{ave} = \frac{\int T dA}{A} \quad (1)$$

Linear interpolation was then used to determine temperature of the structure between hourly readings.

For all four modes, a general trend of decreasing natural frequency as temperature increases. The natural frequencies of modes 1 through 4 decreased by 5.57%, 5.45%, 3.21%, and 2.51% (per °C) on average respectively with temperature varying from -24°C to 37°C. Modes 1 and 2 appear to show a bilinear behavior between temperature and natural frequency with a change in slope at a structural temperature of approximately 10°C. The mode 4 frequency appears to decrease nearly linearly as temperature increases for all temperatures. The mode 3 frequency exhibits a bilinear behavior with temperature, however with a distinct non-linear region from -5°C to 5°C.

While the bilinear behavior of modes 1 and 2 is not a new observation, the previous logic behind the phenomenon does not apply in this case. The bilinear relation between natural frequency and temperature observed on the Z24 bridge in Switzerland was attributed to the stiffness of the asphalt wearing surface below freezing as compared to above freezing, with the change in slope occurring very near 0°C (Peeters and De Roeck 2001). However the change in slope for modes 1 and 2 of the I-35W St. Anthony Falls Bridge occur at a temperature other than freezing and there is no asphalt wearing surface, which suggests the bilinear relationship between natural frequency and temperature exhibited is a result of a different phenomenon.

Of the four modes investigated, mode 1 had the greatest percent variation in natural frequency as temperature varied, decreasing by 5.57% (per °C). Assuming the natural frequency is proportional to the square root of the stiffness divided by the mass of the structure, a 5.57% change in natural frequency corresponds to an approximately 11% change in modal stiffness assuming all other variables remain constant. While an 11% change in stiffness is large, it is theoretically conceivable. However, the percent change in natural frequency of the first mode when considering the absolute maximum and minimum frequencies identified was approximately 25%. The large spread of natural frequencies identified can be seen in Figures 19 through 22. Assuming only stiffness contributed to the change in

natural frequency, and the structure remained undamaged throughout the data collection period, this would represent an approximately 43.75% change in modal stiffness. This clearly illustrates that factors other than modulus of elasticity and stiffness alone contribute to changes in natural frequency of a structure.

CHAPTER 8: TEMPERATURE GRADIENT EFFECTS

To further understand temperature effects on natural frequency, both temporal and spatial thermal gradients were investigated. Temporal gradients were taken as the hourly rate of change in the average structural temperature at midspan, and spatial gradients were taken as the first centroidal moment of temperature as calculated by Eqn. (2), where I is the moment of inertia of the cross section and $\bar{Y}$ is the centroid of the cross section. Similar to average structural temperature (Eqn. (1)), the spatial temperature gradient was calculated at midspan of the river span. The same thermistors utilized for average structural temperature calculations were utilized for spatial temperature gradient calculations (Table 4).

$$\Delta_{sp} = \frac{\int (y - \bar{Y}) T dA}{I} \quad (2)$$

Gradients were calculated when the average temperature of the structure was 16°C, as this was the temperature that occurred the greatest number of times over the period of recorded data. At an average structural temperature of 16°C, modes 1 through 4 were identified 790 times, 837 times, 613 times, and 310 times respectively.

For spatial gradients, no relationship between gradient and natural frequency was identified. Figures 23 through 26 show the effect of spatial gradient on each of the first four natural frequencies when the average temperature of the structure is 16°C. The linear regression line appears to suggest that as spatial gradient increases for a fixed structural temperature, the natural frequency increases slightly, however the slope of the line is dominated by a few points above $\Delta_{sp} \approx .07^\circ\text{C}$. Similar behavior was seen for other temperatures as well. Given the uniquely large dataset, the lack of a clear relationship suggests that one does not exist.

The effect of temporal gradient on natural frequency was similar to that of spatial gradient. Figures 27 through 30 show the relationship between each of the first four natural frequencies and the magnitude of the temporal structural temperature gradient at an average structural temperature of 16°C. The linear regression line as plotted shows a very slight positive correlation, however the relationship is skewed by outliers at temporal gradients greater than 2°C per hour. Looking only at gradients below 2°C per hour shows that there is no significant relationship between temporal gradient and natural frequency.

CHAPTER 9: SUMMARY AND CONCLUSIONS

The vibration data from the I-35W St. Anthony Falls Bridge offers a unique opportunity to investigate the environmental effects on modal parameters of a concrete bridge. Not only does the dataset span many years, but the location of the bridge, Minneapolis, Minnesota, is one of weather extremes. The deployment of $\pm 2g$ DC accelerometers on the bridge meant that additional work was required to accurately and practically identify the modal parameters of interest using NExT-ERA/DC.

The primary challenge associated with the quality and quantity of vibration data was the detection of information-rich sections of data. It was found that the ratio of peak signal amplitude to signal RMS was a strong indicator of a good signal. A large peak-to-RMS ratio indicated a large excitation event followed by free vibration, whereas peak and RMS as signal quality indicators alone were unable to differentiate between instances of ambient free vibration and forced vibration.

Additionally, careful selection of reference channels was required. The selection of individual reference channels at locations of maximum relative displacement for each target mode was deemed appropriate. The addition of a combination of reference channels, which were excited under all target modes, increased the robustness of the NExT-ERA/DC routine. A novel sorting technique was also developed to facilitate the integration of the results found using the various reference channels as well as to efficiently sort the results by mode shape.

Using NExT-ERA/DC and the developed signal detection routine, 29,333 data segments were analyzed with success rates of 81.95%, 89.94%, 63.45%, and 38.88% for the first four modes, respectively. From this large set of results, it was determined that generally as temperature increases, the natural frequency of the first four modes decreases; however, mode 3 exhibited contradictory behavior as the temperature crossed 0°C. Modes 1 and 2 also exhibited a bilinear behavior, whereas the fourth natural frequency appeared to decrease linearly at a constant rate as temperature increased.

Based on the variation of the natural frequencies with temperature, the following conclusions can be drawn.

- Similar to previous observations, natural frequency appears to generally decrease as temperature increases; however, as demonstrated by the behavior of Mode 3, this rule is not universally true.
- Assuming all vibration data was recorded while the bridge was in an undamaged state, the magnitude of change in natural frequency as temperature varies is too great to attribute to change in the modulus of elasticity alone. Other factors, such as humidity, which are correlated with temperature, must also be contributing to the variation in natural frequency.
- The bilinear behavior of the first and second natural frequency with temperature cannot be attributed to asphalt, as there was no wearing surface on the bridge during data collection. As such, another phenomenon must be responsible for the bilinear behavior of this bridge
- Both temporal and spatial temperature gradients do not appear to have a significant effect on the natural frequencies of the structure.

TABLES

Table 1: Assumed Displacement Vectors

Mode	1	2	3	4
Accel. #				
SP1 Ext	-1	1	1	-1
SP1 Int	-1	1	1	1
ACC7	1	-1	1	1
ACC6	1	0	1	1
ACC2	1	1	1	1
ACC1	1	1	1	1
SP2 Ext	1	1	1	1
SP2 Int	1	1	1	-1
ACC4	1	1	0	1
ACC5	1	1	0	1
ACC13	1	1	-1	1
ACC11	1	1	-1	1
ACC12	1	1	-1	0
SP3 Ext	-1	-1	1	-1
SP3 Int	-1	-1	1	1

Table 2: Success rate as a function of Peak Signal and RMS

Peak Signal (g)	Value		0.0023-0.0058	0.0058-0.0093	0.0093-0.0128	0.0128-0.0163	0.0163-0.0198	0.0198-0.0233	0.0233-0.0268	0.0268-0.0303	0.0303-0.0338
	Instances		3	143	510	386	131	31	13	4	3
	Success Rate	Mode 1	0.00%	37.06%	11.18%	5.70%	5.34%	0.00%	0.00%	0.00%	0.00%
		Mode 2	0.00%	56.64%	27.84%	20.98%	13.74%	9.68%	15.38%	0.00%	0.00%
		Mode 3	0.00%	10.49%	13.33%	11.14%	6.11%	16.13%	0.00%	0.00%	0.00%
		Mode 4	66.67%	54.55%	51.18%	40.41%	30.53%	22.58%	7.69%	25.00%	0.00%
Signal RMS	Value		0-0.0007	0.0007-0.0014	0.0014-0.0021	0.0021-0.0028	0.0028-0.0035	0.0035-0.0042	0.0042-0.0049	0.0049-0.0056	0.0056-0.0063
	Instances		2	79	444	371	185	84	31	17	7
	Success Rate	Mode 1	0.00%	50.63%	15.54%	5.66%	2.16%	5.95%	0.00%	0.00%	0.00%
		Mode 2	0.00%	59.49%	34.01%	23.99%	14.59%	10.71%	3.23%	17.65%	0.00%
		Mode 3	0.00%	12.66%	13.96%	9.97%	10.27%	9.52%	9.68%	0.00%	0.00%
		Mode 4	50.00%	63.29%	55.18%	40.43%	35.14%	25.00%	22.58%	29.41%	28.57%
Peak/RMS Ratio	Value		1-2.	2-3.	3-4.	4-5.	5-6.	6-7.	7-8.	8-9.	9-10.
	Instances		0	5	72	281	432	320	103	9	2
	Success Rate	Mode 1	0.00%	20.00%	1.39%	3.56%	12.96%	14.69%	23.30%	0.00%	0.00%
		Mode 2	0.00%	0.00%	11.11%	21.00%	28.01%	33.13%	31.07%	11.11%	0.00%
		Mode 3	0.00%	40.00%	11.11%	7.83%	12.73%	12.50%	10.68%	11.11%	0.00%
		Mode 4	0.00%	80.00%	30.56%	36.30%	42.59%	50.94%	62.14%	55.56%	100.00%

Table 3: Identified Natural Frequencies and Damping

Mode	Fn (Hz)	Coef. Var.	ζ	Coef. Var.	% Found	Fn (Hz) (F.E.A)
1	0.82	1.5%	0.02	32.8%	81.9%	0.750
2	1.54	1.9%	0.01	48.3%	89.9%	1.464
3	2.27	1.1%	0.02	42.8%	63.5%	2.143
4	2.35	0.8%	0.01	34.5%	38.9%	2.424

Table 4: Name and location of thermistors utilized in temperature calculations

Thermistor	Gage #	Box	X (in)	Y (in)
Surface	N.A.	Ext.	-149	0
TSETA001	TH_T(32)	Ext.	-149	-2.4857
TSETA002	TH_T(33)	Ext.	-149	-4.0357
TSETA003	TH_T(34)	Ext.	-149	-6.0357
TSETA004	TH_T(35)	Ext.	-149	-7.1607
TSETA005	TH_T(36)	Ext.	-149	-8.5357
TSETA006	TH_T(37)	Ext.	-149	-10.161
TSEWA001	TH_T(23)	Ext.	-135.96	-36.92
TSEWB002	TH_T(24)	Ext.	-125.49	-81
TSEWC001	TH_T(21)	Ext.	-115.68	-123.25

FIGURES

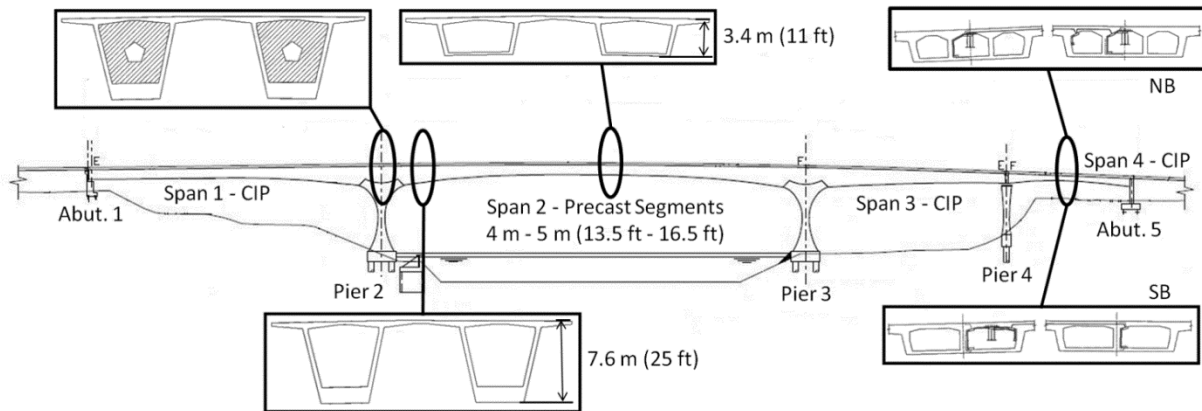
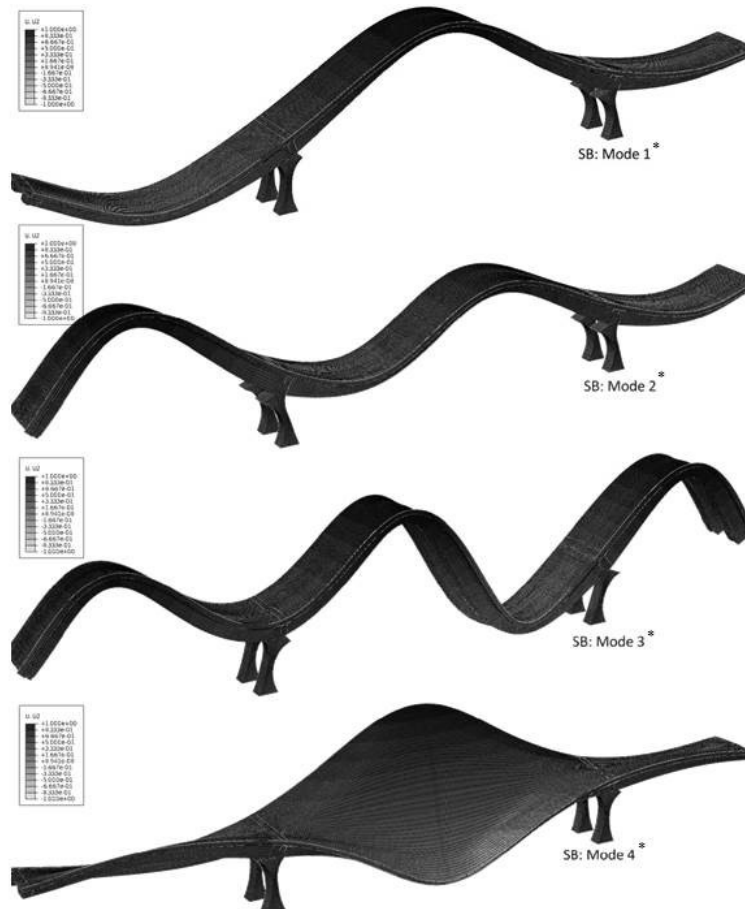


Figure 1: Elevation looking west of the new I-35W St. Anthony Falls Bridge



*French et al. "Instrumentation, Monitoring, and Modeling of the I-35W Bridge." 2012

Figure 2: Southbound Estimated Mode Shapes 1 Through 4 as Extracted from F.E.A.

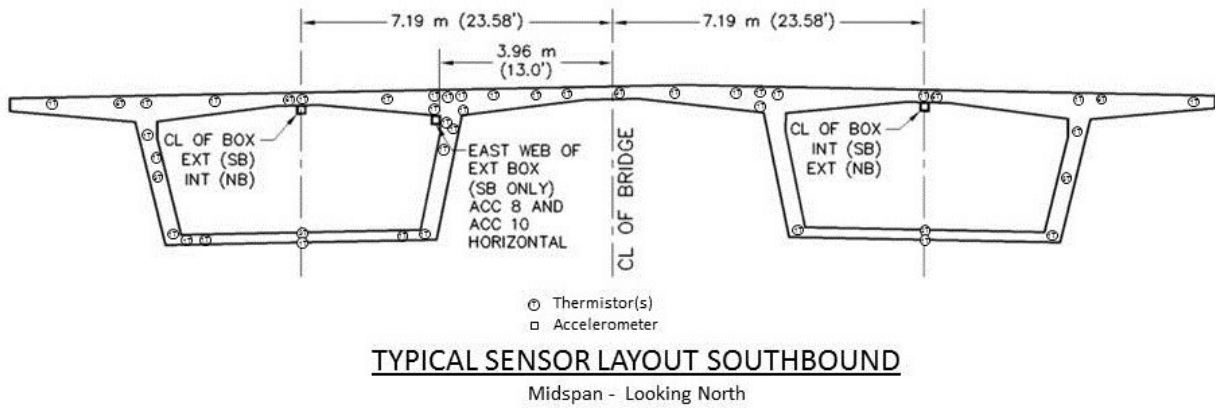


Figure 3: Midspan Span 2 Sensor Layout, Looking North

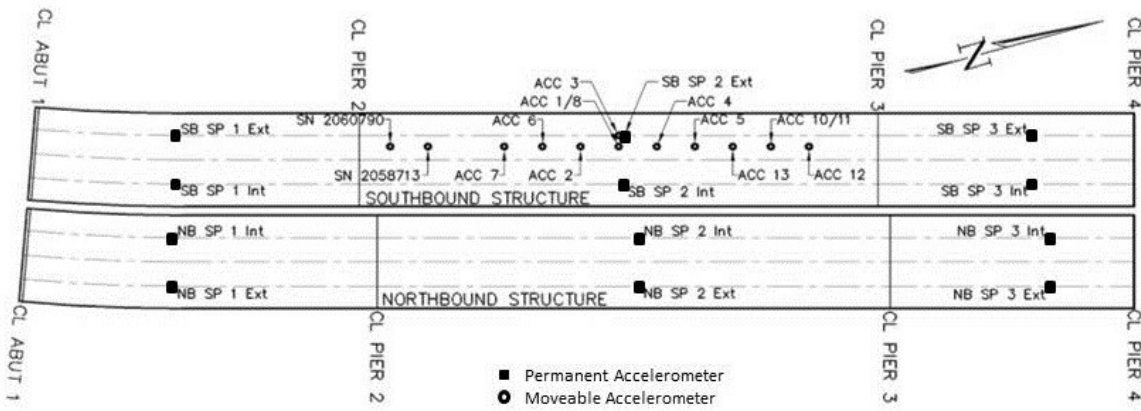


Figure 4: Accelerometer Layout, Plan View

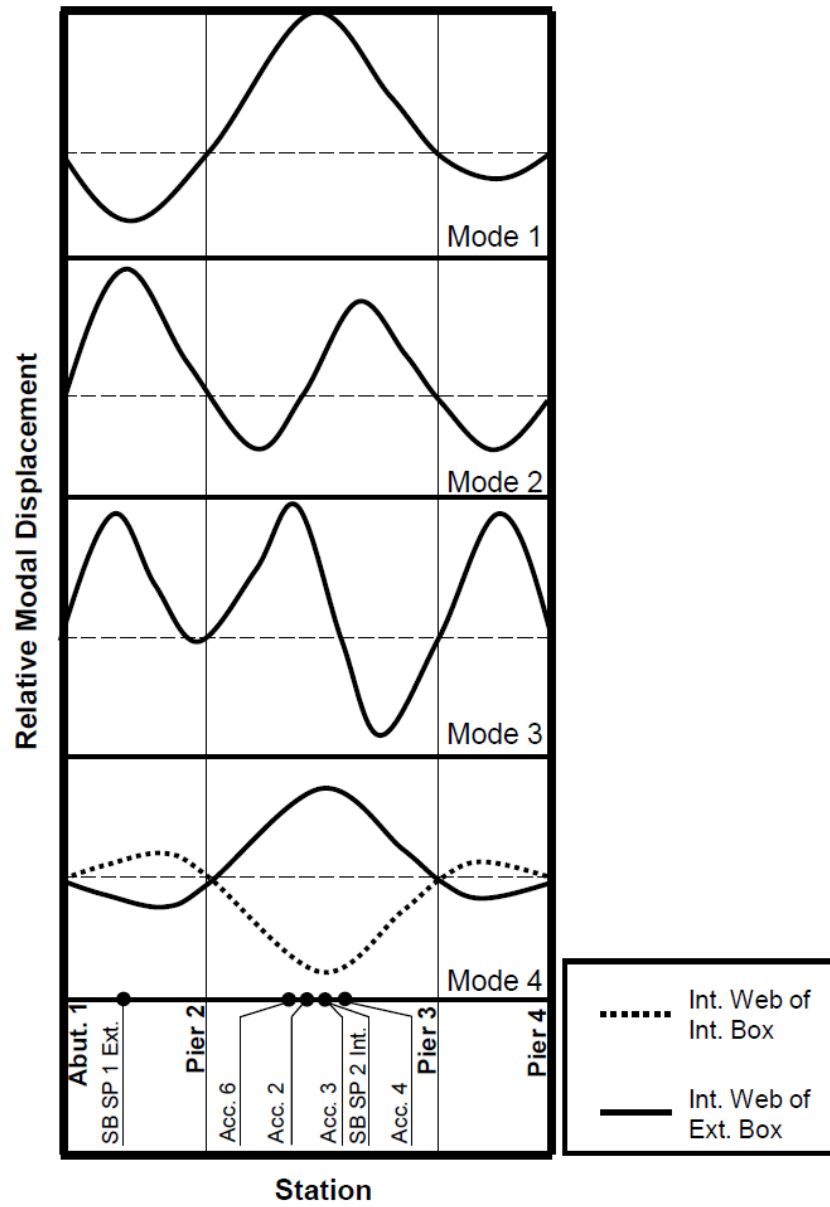


Figure 5: Reference Channel Location with respect to F.E.A. Mode Shape Results

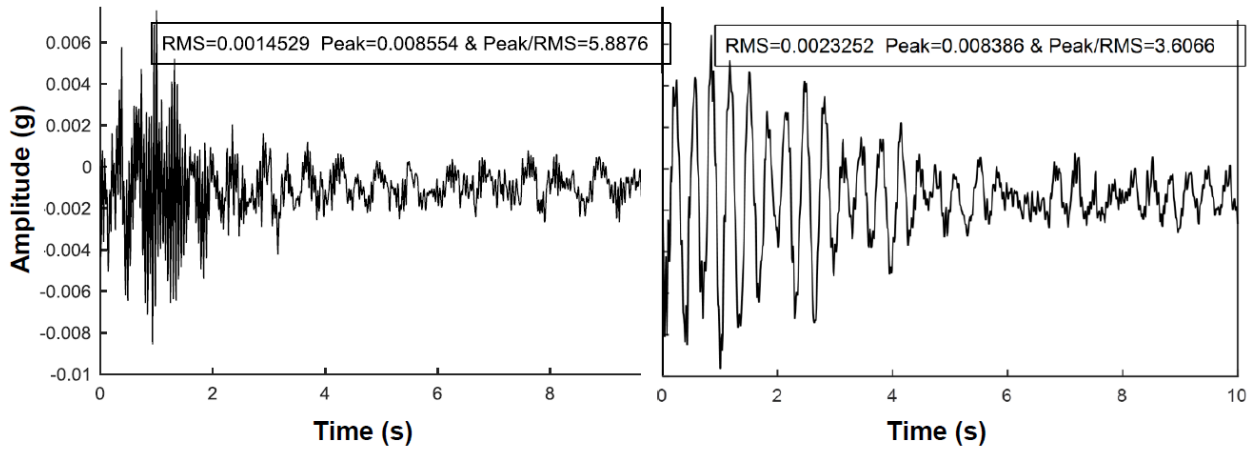


Figure 6: Peak-to-RMS Ratio of Sample Vibration Signals

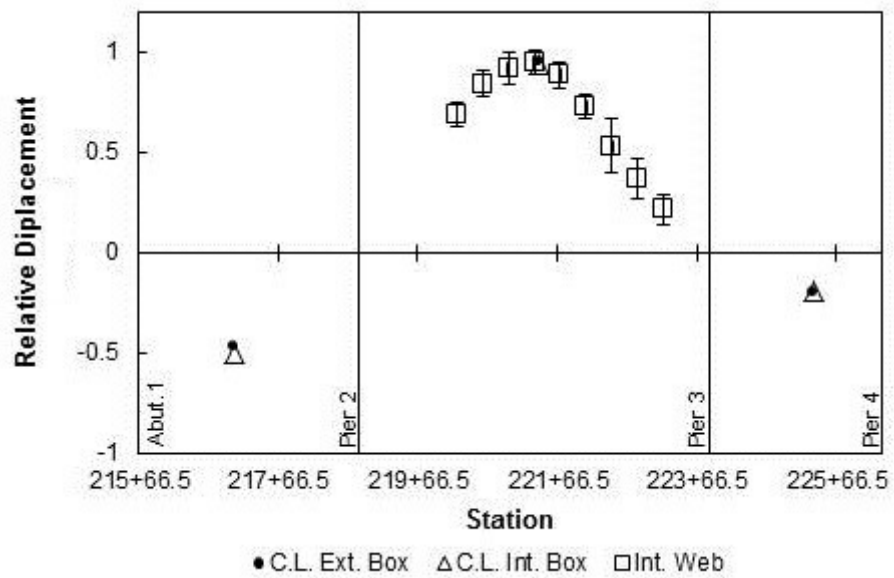


Figure 7: Identified mode Shape of the First Mode

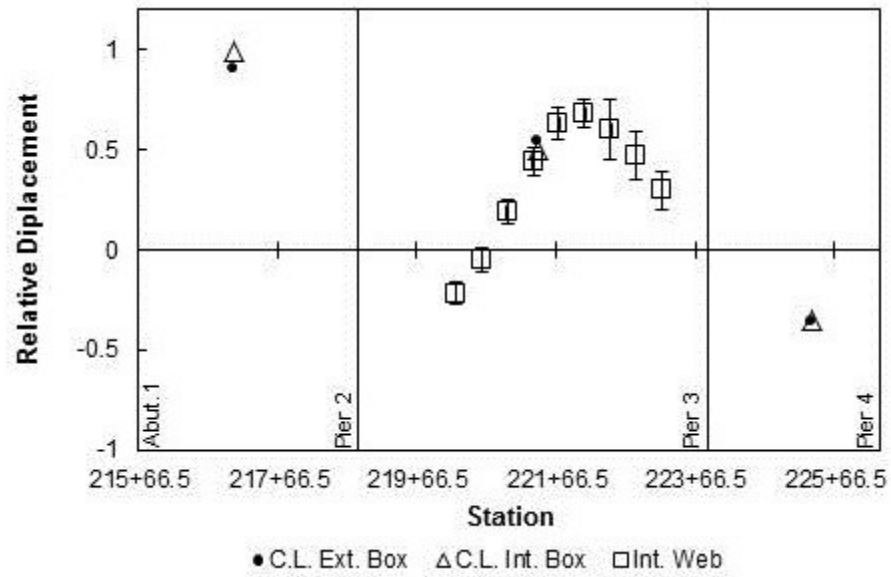


Figure 8: Identified mode Shape of the Second Mode

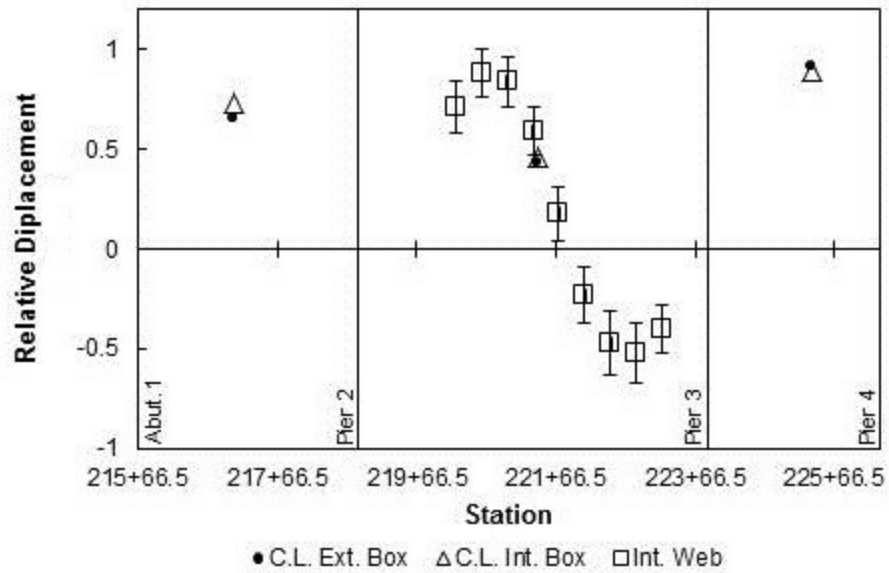


Figure 9: Identified mode Shape of the Third Mode

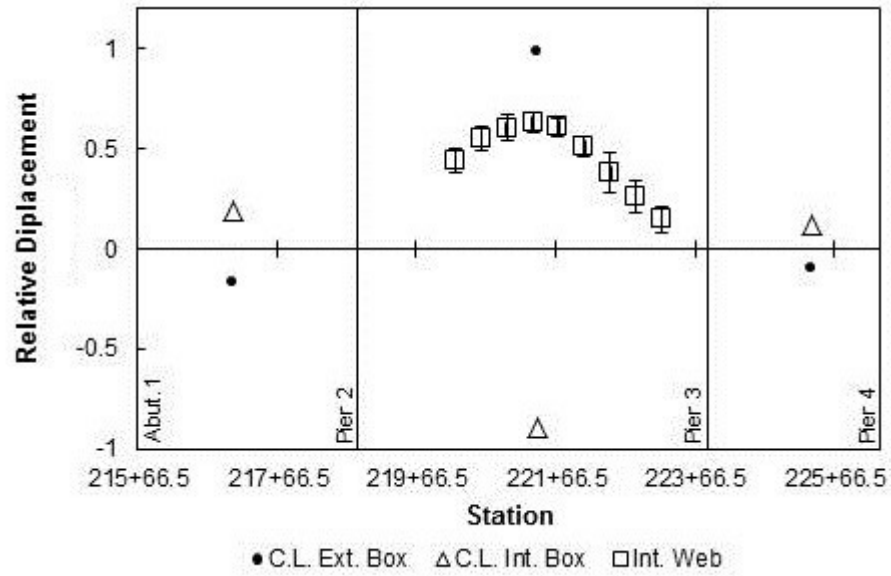


Figure 10: Identified mode Shape of the Fourth Mode

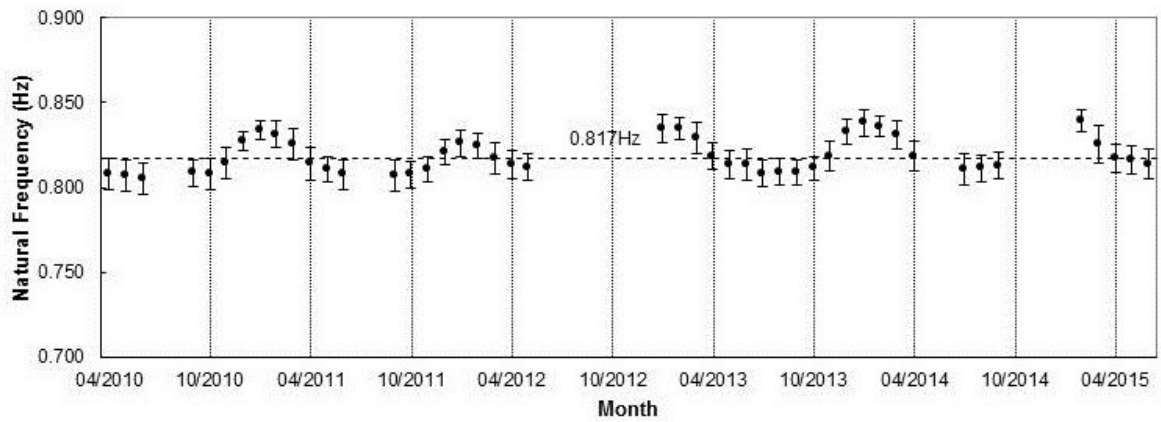


Figure 11: Monthly Average Natural Frequency of the First Mode

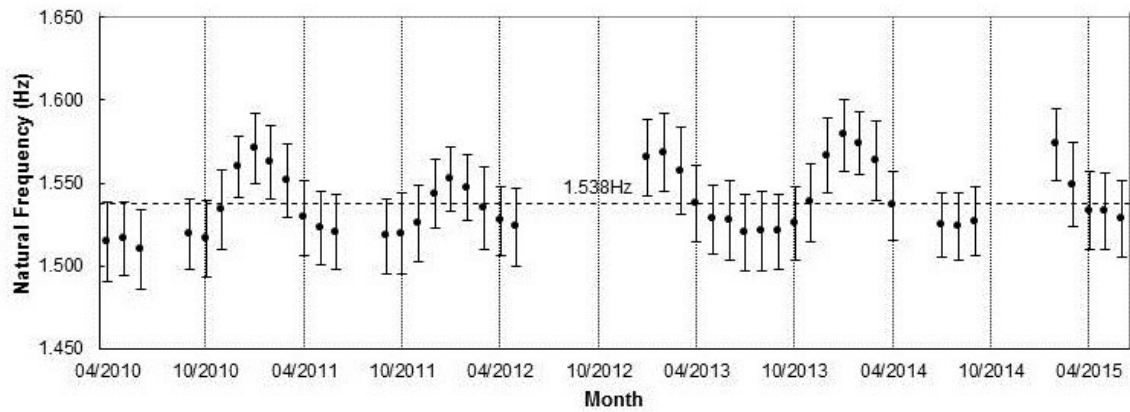


Figure 12: Monthly Average Natural Frequency of the Second Mode

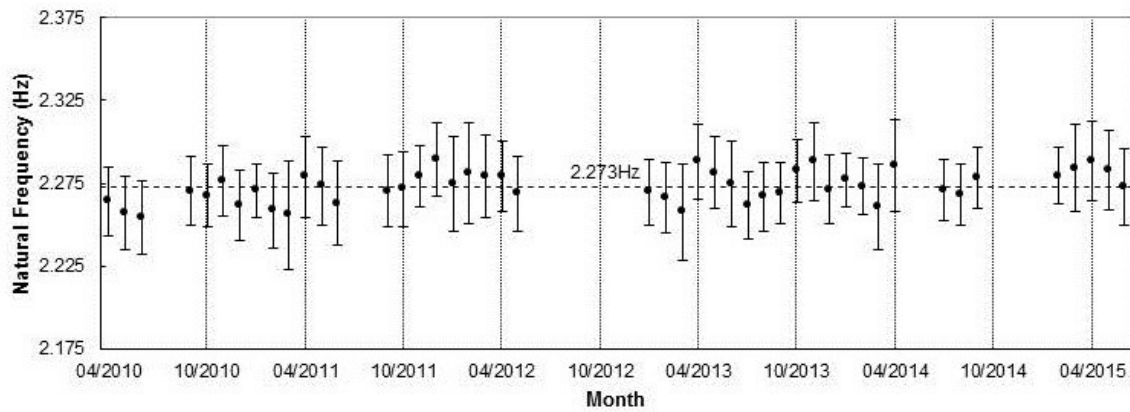


Figure 13: Monthly Average Natural Frequency of the Third Mode

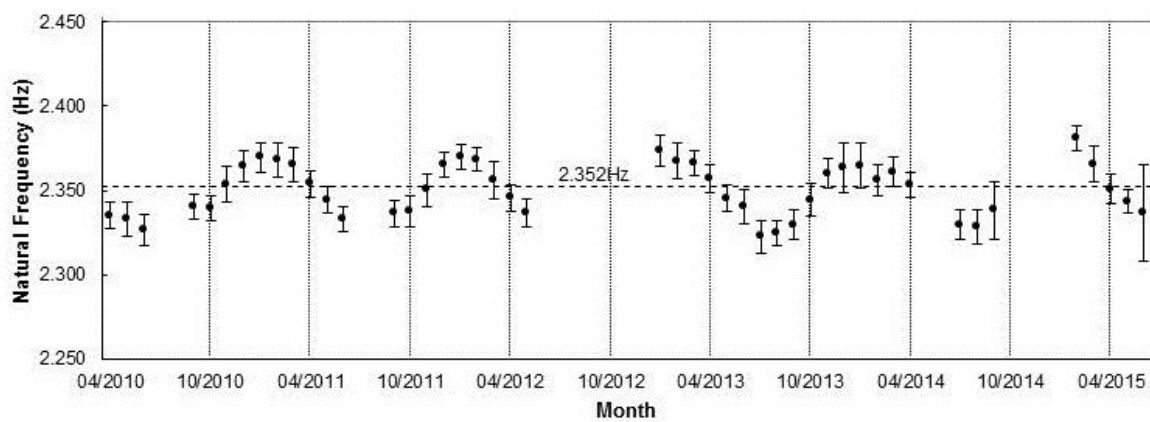


Figure 14: Monthly Average Natural Frequency of the Fourth Mode

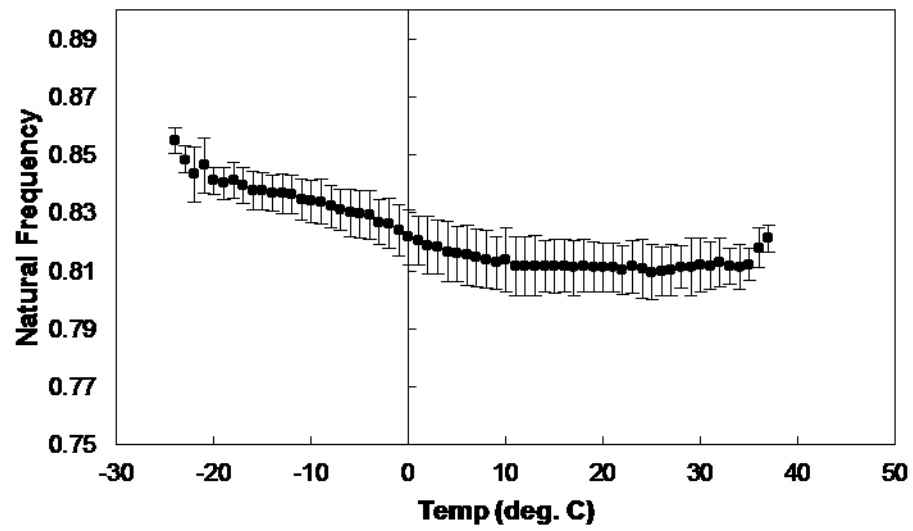


Figure 15: Variation of the First Natural Frequency with Temperature

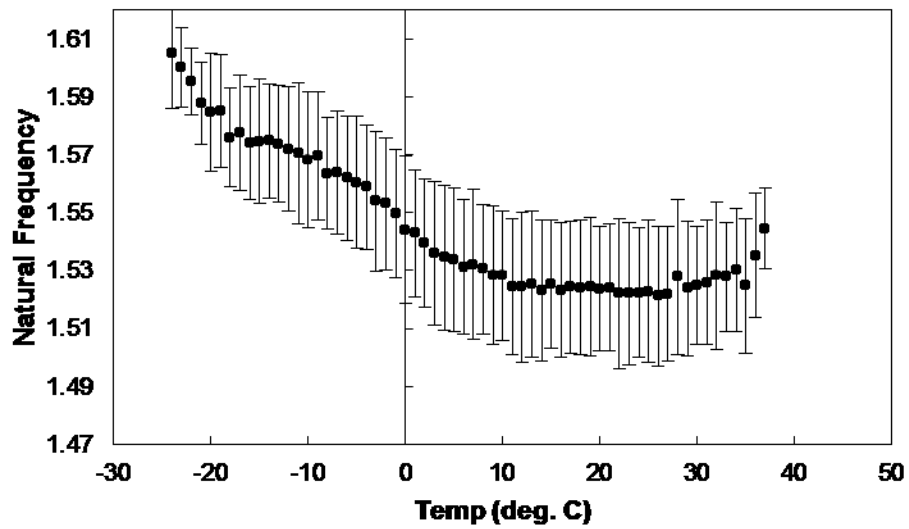


Figure 16: Variation of the Second Natural Frequency with Temperature

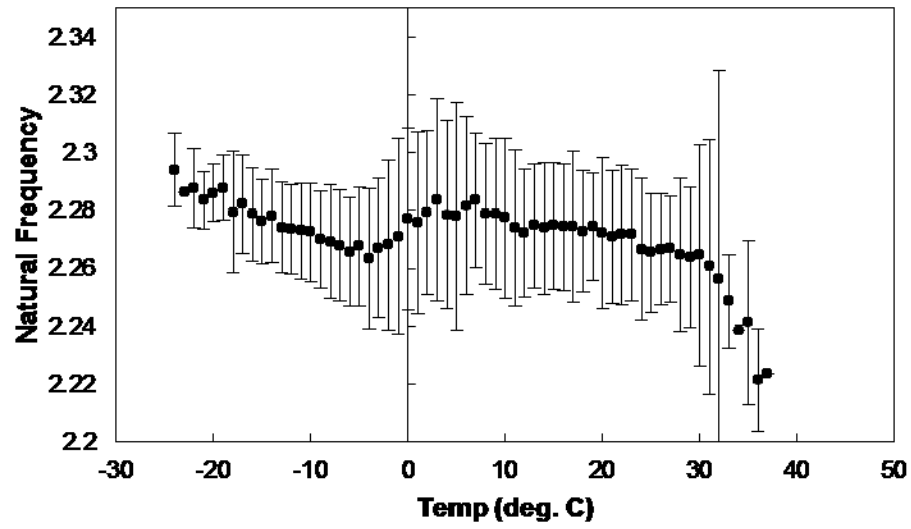


Figure 17: Variation of the Third Natural Frequency with Temperature

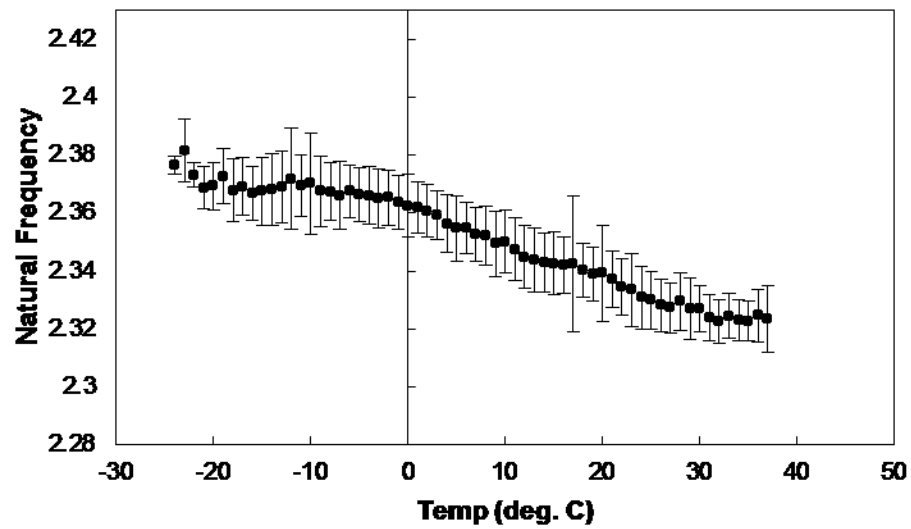


Figure 18: Variation of the Fourth Natural Frequency with Temperature

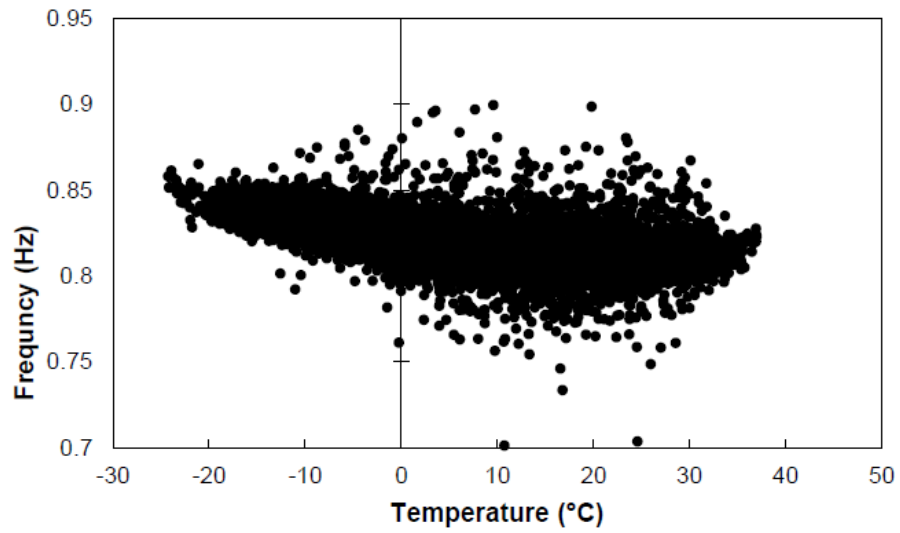


Figure 19: Variation in First Natural Frequency with Temperature

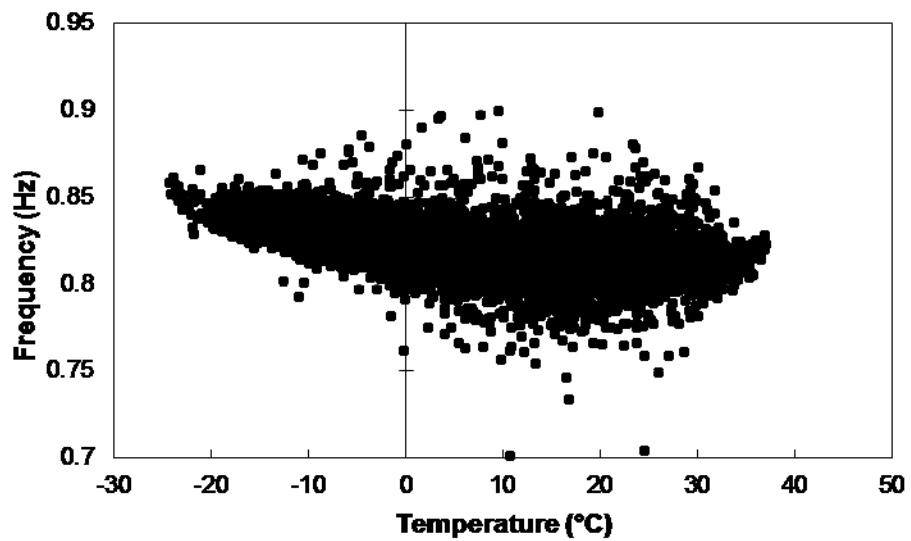


Figure 20: Variation in Second Natural Frequency with Temperature

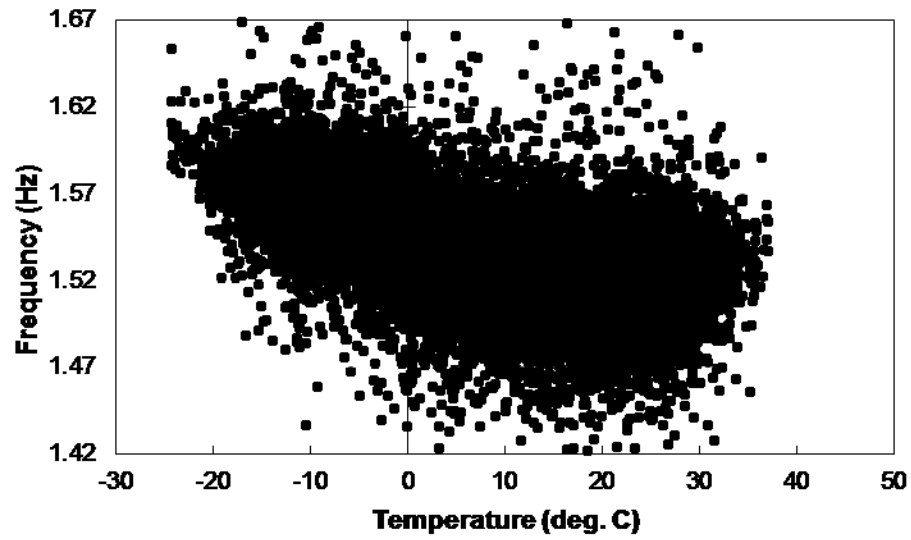


Figure 21: Variation in Third Natural Frequency with Temperature

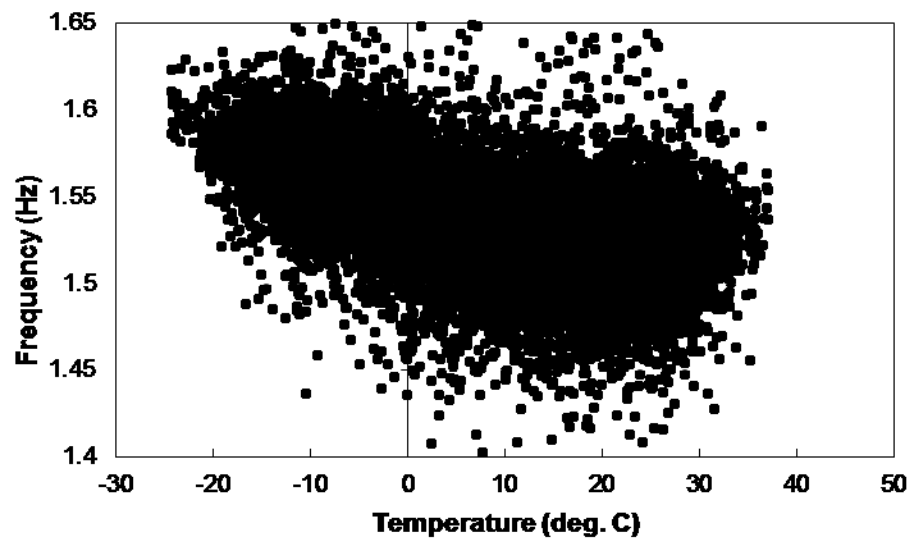


Figure 22: Variation in Fourth Natural Frequency with Temperature

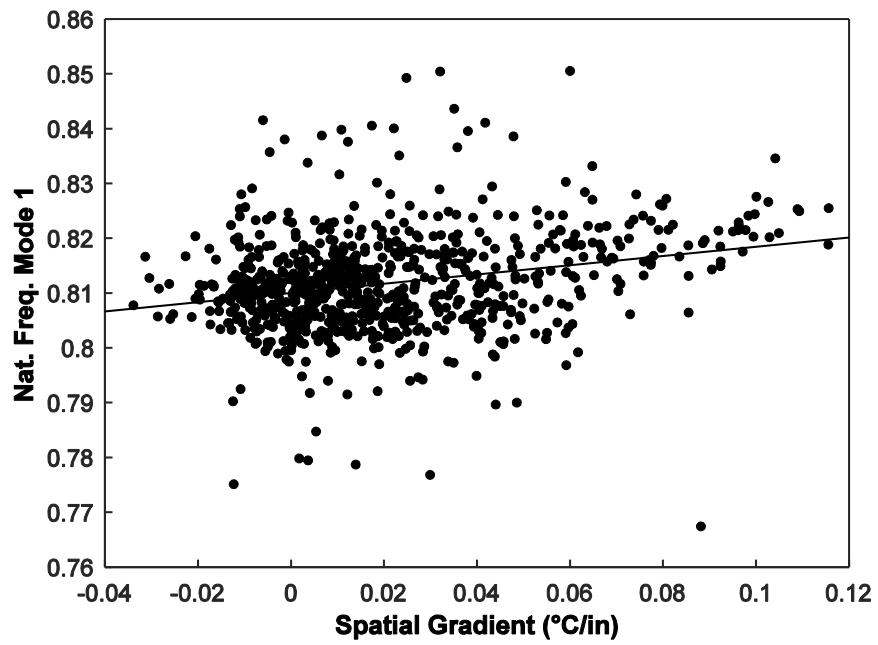


Figure 23: Effects of Spatial Gradient on the First Natural Frequency

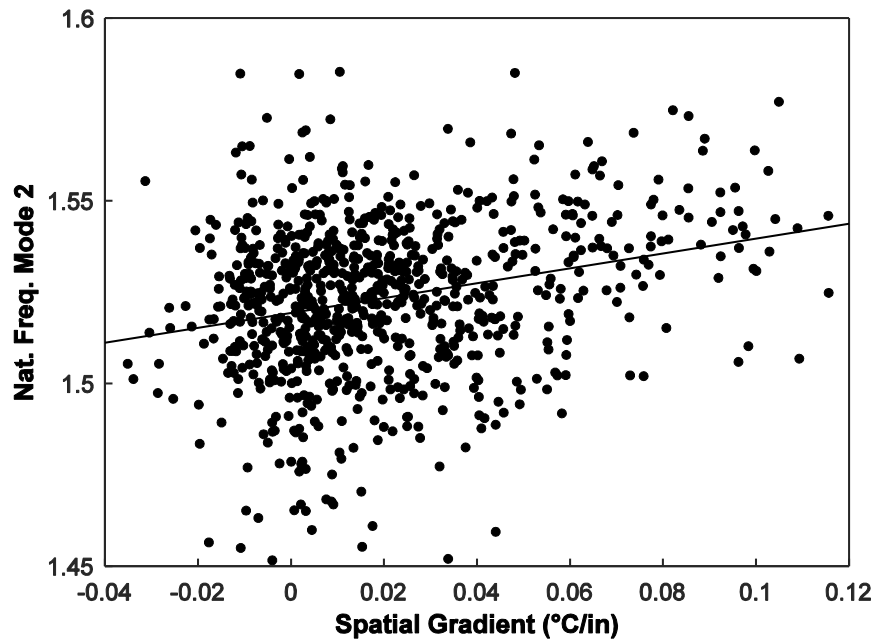


Figure 24: Effects of Spatial Gradient on the Second Natural Frequency

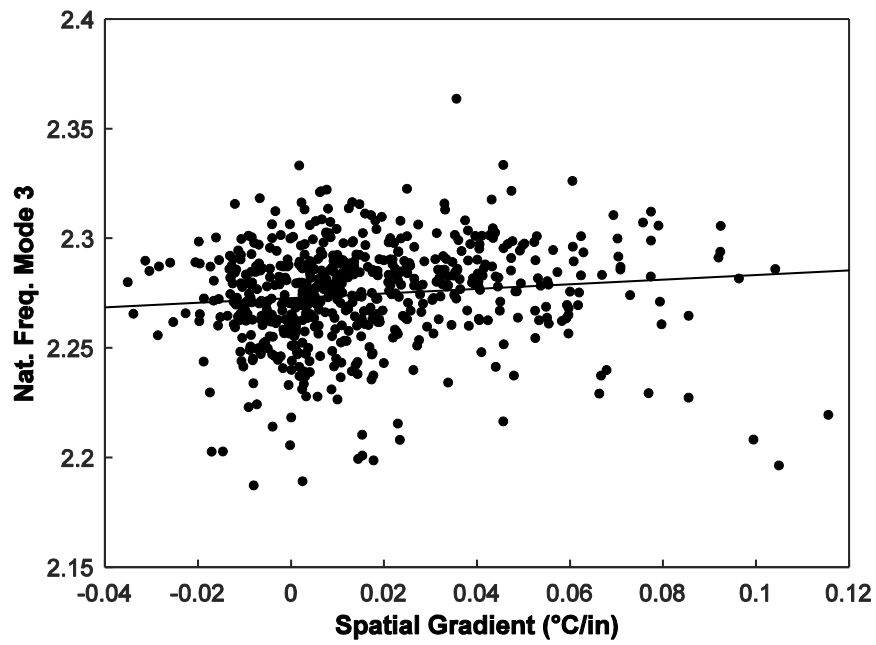


Figure 25: Effects of Spatial Gradient on the Third Natural Frequency

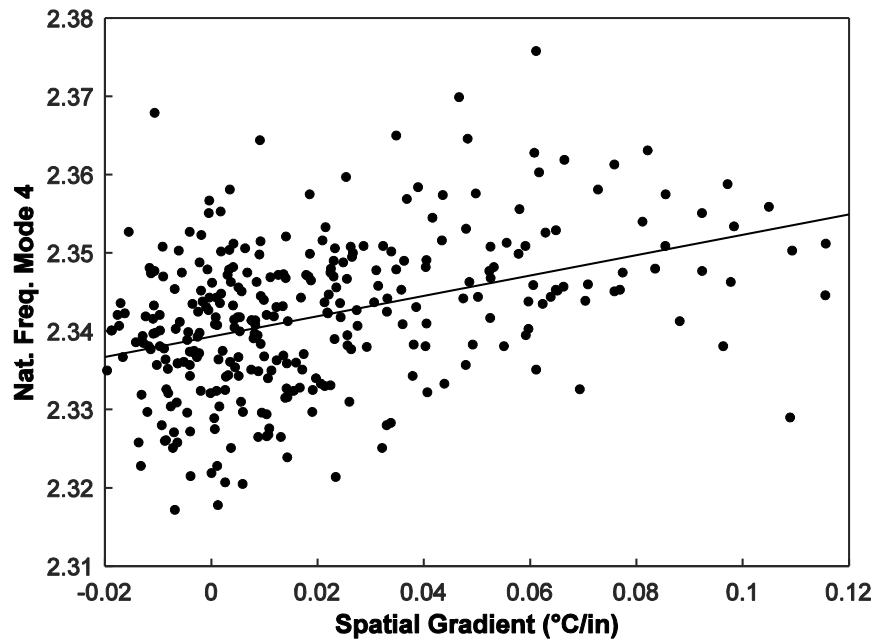


Figure 26: Effects of Spatial Gradient on the Fourth Natural Frequency

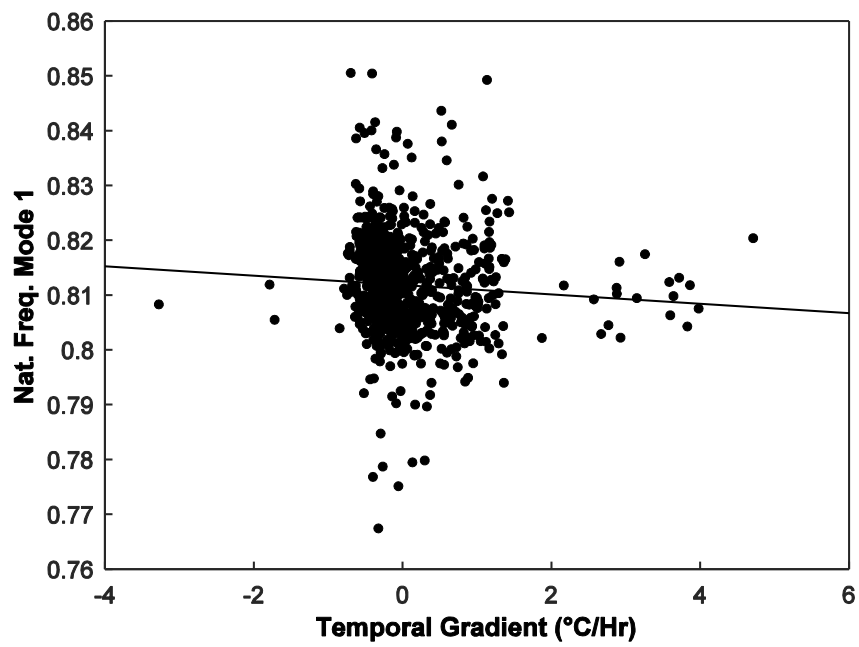


Figure 27: Effects of Temporal gradient on the First Natural Frequency

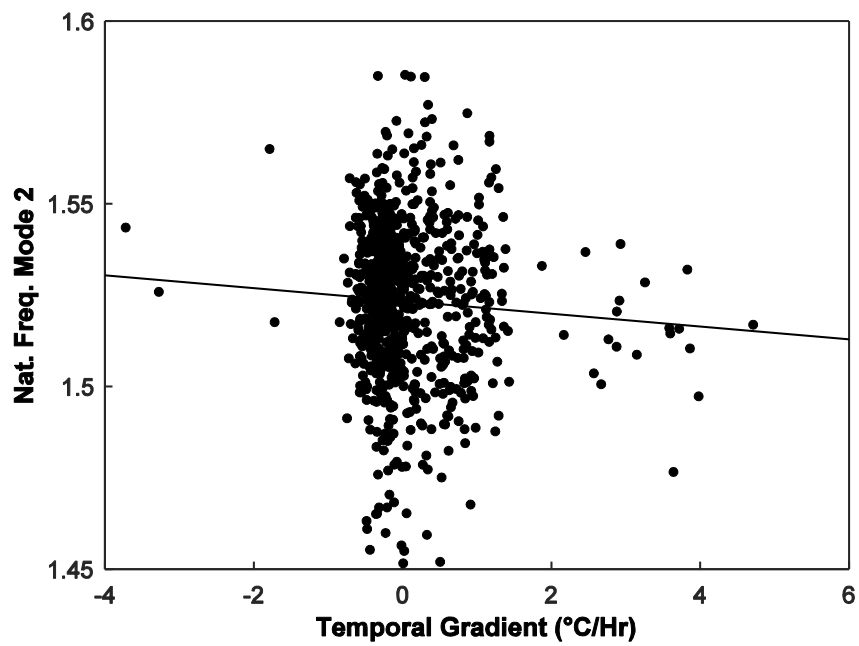


Figure 28: Effects of Temporal gradient on the Second Natural Frequency

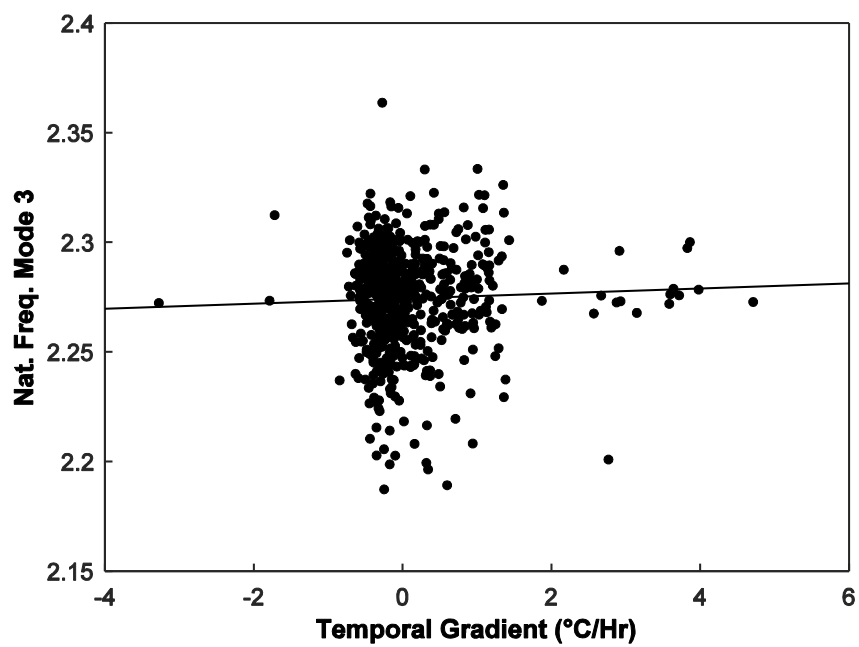


Figure 29: Effects of Temporal gradient on the Second Natural Frequency

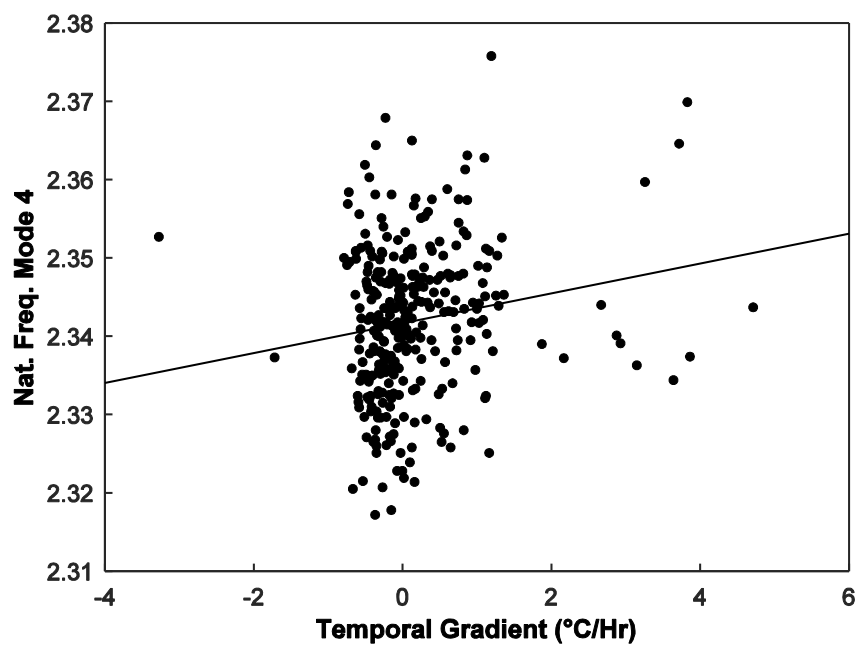


Figure 30: Effects of Temporal gradient on the Second Natural Frequency

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APPENDIX A – ACCELEROMETER OUTAGES

The entire system (northbound and southbound accelerometers) has gone down on three occasions:

- 8/15/2009 to 9/28/2009 – approximately 1.5 months
- 6/25/2011 to 8/28/2010 – approximately 2 months
- 5/27/2012 to 1/10/2013 – approximately 7.5 months

On five other occasions, only the southbound accelerometers have gone down:

- 8/28/2010 to 9/19/2010 – approximately 1 month
- 6/25/2011 to 9/13/2011 – approximately 3 months
- 6/21/2013 to 7/8/2013 – approximately 0.5 months
- 4/27/2014 to 7/12/2014 – approximately 1.5 months
- 7/5/2015 to 7/18/2015 – approximately 0.5 months

Outages appear to occur after electrical storms in the area, and have been attributed to a possibly weak ground in the system, and steps have been taken to rectify the issue as much as possible (French et al. 2012). The accelerometers are wired to collection nodes in series. As the southbound nodes are later in the series, they are more susceptible to outages when a collection node goes down.

APPENDIX B – NEXT-ERA/DC

The NExT methodology was introduced by James et al. (1993). The purpose of this technique is to be able to analyze system dynamic behavior with only immeasurable ambient forcing. The motion of a linear dynamic system is governed by the equation

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} = \mathbf{F}(t) \quad (\text{B.1})$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ the damping matrix, $\mathbf{K}$ the stiffness matrix, $\mathbf{X}$ the displacement of the system, and the $\mathbf{F}(t)$ the external forcing. The displacement and forcing are assumed to be wide-sense stationary random processes. Multiplying both sides of Eq. (C.1) by the displacement vector of a reference degree of freedom, $\mathbf{X}_{REF}$, at time $t-\tau$ and take the expectation operator $E[\cdot]$, yields

$$\begin{aligned} \mathbf{M}E[\ddot{\mathbf{X}}(t)\mathbf{X}_{REF}(t-\tau)] + \mathbf{C}E[\dot{\mathbf{X}}(t)\mathbf{X}_{REF}(t-\tau)] + \mathbf{K}E[\mathbf{X}(t)\mathbf{X}_{REF}(t-\tau)] \\ = E[\mathbf{F}(t)\mathbf{X}_{REF}(t-\tau)] \end{aligned} \quad (\text{B.2})$$

The cross correlation of a wide-sense stationary process is defined by

$$\mathbf{R}_{XX_{REF}}(\tau) = E[\mathbf{X}(t)\mathbf{X}_{REF}(t-\tau)] \quad (\text{B.3})$$

Assume the external force is uncorrelated to the motion of the system for $\tau > 0$. Also, the correlation function is a linear operator, so for the wide-sense stationary processes given,

$$E[\mathbf{F}(t)\mathbf{X}_{REF}(t-\tau)] = 0 \quad (\text{B.4})$$

$$\mathbf{M}E[\ddot{\mathbf{X}}(t)\mathbf{X}_{REF}(t-\tau)] = \mathbf{R}_{\ddot{X}X_{REF}}(\tau) = \ddot{\mathbf{R}}_{XX_{REF}}(\tau) \quad (\text{B.5})$$

$$\mathbf{M}E[\dot{\mathbf{X}}(t)\mathbf{X}_{REF}(t-\tau)] = \mathbf{R}_{\dot{X}X_{REF}}(\tau) = \dot{\mathbf{R}}_{XX_{REF}}(\tau) \quad (\text{B.6})$$

Therefore, the correlation function of the displacements fulfill the unforced (homogeneous) system of equations

$$\mathbf{M}\ddot{\mathbf{R}}_{XX_{REF}}(\tau) + \mathbf{C}\dot{\mathbf{R}}_{XX_{REF}}(\tau) + \mathbf{K}\mathbf{R}_{XX_{REF}}(\tau) = 0 \quad (\text{B.7})$$

A similar process can be followed using the correlation for the system's accelerations to yield

$$\mathbf{M}\ddot{\mathbf{R}}_{\ddot{X}\ddot{X}_{REF}}(\tau) + \mathbf{C}\dot{\mathbf{R}}_{\ddot{X}\ddot{X}_{REF}}(\tau) + \mathbf{K}\mathbf{R}_{\ddot{X}\ddot{X}_{REF}}(\tau) = 0 \quad (\text{B.8})$$

The importance of this formulation is that the forcing term is eliminated and the system is written in terms of accelerations only. Therefore, the immeasurable ambient forcing does not need to be known, nor does any accelerometer data need to be integrated to velocity or displacement records.

The ERA is a method proposed by Juang and Pappa (1985, 1986) to extract the modal parameters of a dynamic system. The linear, time-invariant system can be represented in state space for as

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \quad (\text{B.9})$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \quad (\text{B.10})$$

The algorithm starts by constructing a generalized Hankel matrix of the form

$$\mathbf{H}(k-1) = \begin{bmatrix} \mathbf{Y}(k) & \mathbf{Y}(k+1) & \cdots & \mathbf{Y}(k+p) \\ \mathbf{Y}(k+1) & \mathbf{Y}(k+2) & \cdots & \mathbf{Y}(k+p+1) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Y}(k+r) & \mathbf{Y}(k+r+1) & \cdots & \mathbf{Y}(k+p+r) \end{bmatrix} \quad (\text{B.11})$$

where r is the number of block rows and p is the number of block columns. Each $\mathbf{Y}(k)$ is an $n \times m$ block matrix, where n is the number of degrees of freedom (i.e., the number of measurement stations) in the system and m is the number of reference degrees of freedom used to construct the matrix. In the procedure defined by Juang and Pappa, the $\mathbf{Y}(k)$ block matrices are constructed using the free impulse response functions of the system at time increment k . However, using equation B.8 from the NExt method, the block matrices from the correlation functions $\mathbf{R}_{\ddot{X}\ddot{X}_{REF}}(\tau)$ can be formed as follows:

$$\mathbf{Y}(k) = \begin{bmatrix} \mathbf{R}_{1,1}(k) & \mathbf{R}_{1,2}(k) & \cdots & \mathbf{R}_{1,m}(k) \\ \mathbf{R}_{2,1}(k) & \mathbf{R}_{2,2}(k) & \cdots & \mathbf{R}_{2,m}(k) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{R}_{n,1}(k) & \mathbf{R}_{n,2}(k) & \cdots & \mathbf{R}_{n,m}(k) \end{bmatrix} \quad (\text{B.12})$$

for which $\mathbf{R}_{ij}(k)$ refers to the correlation between the i th degree of freedom and the j th reference DOF at time step k .

After deciding on the reference degrees of freedom, we begin by computing the cross-correlations for all degrees of freedom against the reference DOFs. These are assembled in Hankel matrices $\mathbf{H}(0)$ and $\mathbf{H}(1)$. The singular value decomposition of $\mathbf{H}(0)$ is calculated

$$\mathbf{H}(0) = \mathbf{P}\mathbf{D}\mathbf{Q}^T \quad (\text{B.13})$$

where $\mathbf{D}$ is a diagonal matrix and $\mathbf{P}$ and $\mathbf{Q}$ are both orthogonal. All the terms of $\mathbf{D}$ do not need to be saved. For an ideal system, this matrix would contain $2M$ values along the diagonal arranged in descending order, where M is the total number of modes in the system. However, for a physical system with measurement noise, every term along the diagonal of $\mathbf{D}$ will be filled. The smallest valued entries do not contribute significantly to the system, so it is advisable to remove these by eliminating the last rows and column of $\mathbf{D}$ to reduce it to a $2M \times 2M$ matrix called $\mathbf{D}_r$, where M is the number of modes that will be calculated. Matrices $\mathbf{P}$ and $\mathbf{Q}$ likewise can likewise be reduced to matrices $\mathbf{P}_r$ and $\mathbf{Q}_r$ by keeping only the first $2M$ columns of each. The state transition matrix $\mathbf{A}$ for the dynamic system using the reduced decomposition matrices can be constructed as

$$\mathbf{A} = \mathbf{D}_r^{-1/2} \mathbf{P}_r \mathbf{H}(1) \mathbf{Q}_r \mathbf{D}_r^{-1/2} \quad (\text{B.14})$$

The eigenvalues of $\mathbf{A}$ are given as $\mathbf{Z}$ and the eigenvectors as $\mathbf{\Psi}$. The eigenvalues can be transformed back to the continuous time domain using

$$s_i = \sigma_i \pm j\omega_i = \ln(z_i) / \Delta t \quad (\text{B.15})$$

where Δt is the time increment between samples, σ_i is the damping ratio for the i th mode, and ω_i is the modal frequency of the i th mode. The forcing matrix $\mathbf{B}$ and output matrix $\mathbf{C}$ can be constructed using

$$\mathbf{B} = \mathbf{D}_r^{1/2} \mathbf{Q}_r^T \quad (\text{B.16})$$

$$\mathbf{C} = \mathbf{P}_r \mathbf{D}_r^{1/2} \quad (\text{B.17})$$

These can be used to construct the modal participation factor matrix φ and the mode shapes Φ :

$$\varphi = \Psi^{-1} \mathbf{B} \quad (\text{B.18})$$

$$\Phi = \mathbf{C} \Psi \quad (\text{B.19})$$

In implementation of this method, the number of modes M to extract must be selected prior to calculations. For physical data with noise, the number of modes in the system is theoretically infinite, but practically speaking, only a subset of the modes can be both relevant and calculated accurately. The number of block rows r and block columns p must also be specified. Finally, the reference nodes for the cross-correlation in the NExT must be specified: a subset (or all) of the data channels can be selected simultaneously for the calculation, or can be selected individually if memory limitations are a concern for computations.

APPENDIX C – CONSISTENT MODE INDICATOR

The consistent-mode indicator (CMI) proposed by Pappa et al. (1993) quantifies the temporal consistency of the identified modes. Modes that are consistent with time are typically physical modes that reflect the dynamic system. Modes that are inconsistent with time are likely due to random variations and noise, and can be eliminated from consideration. The CMI for mode i is defined as the product of the extended modal amplitude coherence (EMAC) and the modal phase collinearity (MPC):

$$\text{CMI}_i = \text{EMAC}_i * \text{MPC}_i \quad (\text{C.1})$$

The EMAC quantifies the temporal consistency of the mode shape by comparing the measured mode shape at time T_0 to the predicted mode shape obtained by projecting the mode shape at time $t = 0$ forward. Let $(\Phi_{ij})_0$ be the identified mode shape for mode i at degree of freedom j at time $t = 0$, $(\Phi_{ij})_{T_0}$ be the identified mode shape at time $t = T_0$, and $(\tilde{\Phi}_{ij})_{T_0}$ be the predicted mode shape at time $t = T_0$ calculated by

$$(\tilde{\Phi}_{ij})_{T_0} = (\Phi_{ij})_0 e^{s_i T_0} \quad (\text{C.2})$$

where s_i is the transformed eigenvector for mode i from equation B.15. The ratio of the magnitudes of the predicted and measured modes at $t = T_0$ is given by

$$R_{ij} = \frac{|(\Phi_{ij})_{T_0}|}{|(\tilde{\Phi}_{ij})_{T_0}|} \quad \text{for } |(\Phi_{ij})_{T_0}| < |(\tilde{\Phi}_{ij})_{T_0}| ; \quad \text{otherwise} \quad R_{ij} = \frac{|(\tilde{\Phi}_{ij})_{T_0}|}{|(\Phi_{ij})_{T_0}|} \quad (\text{C.3})$$

The measured and predicted phase angle can also be compared. Define weighting function W_{ij} for mode i and degree of freedom j as

$$W_{ij} = 1 - (|P_{ij}|/(\pi/4)) \quad \text{for } |P_{ij}| \leq \frac{\pi}{4} ; \quad \text{otherwise} \quad W_{ij} = 0 \quad (\text{C.4})$$

where P_{ij} is defined as

$$P_{ij} = \arg\left((\Phi_{ij})_{T_0}/(\tilde{\Phi}_{ij})_{T_0}\right) \quad (\text{C.5})$$

The output EMAC (written as EMAC_{ij}^O) is defined as

$$\text{EMAC}_{ij}^O = R_{ij} * W_{ij} \quad (\text{C.6})$$

The input EMAC for mode i and reference node k , EMAC_{ik}^I , can be calculated using the measured and predicted modal participation factors φ at time $t = T_0$. This is performed in the same manner as shown in equations C.2 through C.6, except substitute φ_{ik} in for all instances of Φ_{ij} . The input-output EMAC values for all modes i , degrees of freedom j , and reference node k can be computed and assembled into weighted nodal EMAC values by the following equation:

$$\text{EMAC}_i = \frac{(\sum_{j=1}^n \text{EMAC}_{ij}^O * (\Phi_{ij})_0^2) (\sum_{k=1}^m \text{EMAC}_{ik}^I * (\Phi_{ik})_0^2)}{(\sum_{j=1}^n (\Phi_{ij})_0^2) * (\sum_{k=1}^m (\Phi_{ik})_0^2)} \quad (\text{C.7})$$

The MPC describes the spatial consistency of the mode. For a given classical dynamic mode, each location of the structure should vibrate exactly in-phase or out-of-phase with all other locations. In order to determine the degree of this “monophase” behavior, calculate the variance and covariance of the real and imaginary parts for each mode shape i to be

$$S_{xx} = \text{re}(\Phi_i)^T \text{re}(\Phi_i) \quad (\text{C.8})$$

$$S_{yy} = \text{im}(\Phi_i)^T \text{im}(\Phi_i) \quad (\text{C.9})$$

$$S_{xy} = \text{re}(\Phi_i)^T \text{im}(\Phi_i) \quad (\text{C.10})$$

The MPC is calculated from the eigenvalues of the variance-covariance matrix. If the mode is perfectly monophase, then only one of the eigenvalues should be non-zero. The eigenvalues of the variance-covariance are given by

$$\lambda_{1,2} = \frac{S_{xx} + S_{yy}}{2} \pm S_{xy} \sqrt{\left(\frac{S_{yy} - S_{xx}}{2S_{xy}} \right)^2 + 1} \quad (\text{C.11})$$

The MPC for mode i is defined as follows:

$$\text{MPC}_i = \left(\frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} \right)^2 \quad (\text{C.12})$$

For each mode, the EMAC and MPC values will vary from zero to one. Thus, the CMI will also vary from zero to one. Modes below a certain CMI threshold are eliminated from consideration.

APPENDIX D – REFERENCE CHANNEL INVESTIGATION

To investigate the ability of NExT-ERA/DC to identify each of the first four modes while reference channels were varied, the NExT-ERA/DC routine was repeatedly implemented with each reference channel used individually. When comparing the results of each NExT-ERA/DC run, mode 1 was found 85% of the time or greater when SB SP 2 Ext, Acc 1, or Acc 6 were used as reference channels. Mode 2 was found 93.85% of the time when AB SO 1 Ext was used as a reference channel. Mode 3 was found 68.31% of the time when SB SP 1 Int was used as a reference channel, and 64.31% of the time when SB SP 1 Ext was used. Mode 4 was found 45.23% of the time when Acc4 was used as a reference channel, and 44.92% of the time when SB SP 2 Ext was used. The complete results of this investigation are shown in Table 1.D.

Table 1.D – Success rate of NExT-ERA/DC as a function of reference channel selection

Mode	Reference channel							
	SP 1 Ext	SP 1 Int	SP 3 Ext	SP 3 Int	SP 2 Ext	SP 2 Int	ACC 1	ACC 2
1	77.54%	75.69%	40.31%	40.62%	85.23%	80.62%	85.54%	84.00%
2	93.85%	92.69%	90.15%	91.08%	91.08%	91.77%	91.08%	72.92%
3	64.31%	68.31%	63.38%	64.92%	40.00%	37.85%	55.69%	64.31%
4	23.08%	16.92%	13.54%	8.62%	44.92%	42.77%	39.38%	38.15%
Mode	ACC 4	ACC 5	ACC 6	ACC 7	ACC 11	ACC 12	ACC 13	Combo-1*
1	82.46%	82.46%	85.54%	80.92%	63.69%	52.62%	71.38%	84.00%
2	91.08%	92.31%	24.62%	82.46%	90.46%	89.85%	90.77%	91.38%
3	16.00%	28.62%	63.08%	61.54%	55.08%	57.54%	48.62%	57.23%
4	45.23%	13.69%	34.77%	32.62%	29.23%	23.08%	34.77%	44.00%

*Acc2, Acc3, and Acc 4

Based on the results of the investigation SB SP 1 Ext, SB SP 2 Ext, and Acc 6 were chosen as reference channels to target the first four modes. It was determined that SB SP 1 Ext as a reference channel was sufficient to find both modes 2 and 3, and that SB SP 2 Ext as reference channel was sufficient for mode 4. Mode 4 was identified at a marginally greater rate with Acc 4 as a reference channel; however mode 3 was identified a much greater rate when using SB SP 2 Ext.

In this channel combination, the reference channel best suited for each mode was not necessarily selected. To account for this as well as the variation in results when a combination of reference channels is used, the combination of Acc 2, Acc 3, and Acc 4 as reference channels was also investigated. While the use of more than 3 reference channels marginally increased the ability of NExT-ERA/DC to identify the first four modes, the increase in required computation time was not warranted.