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Geostatistics for Subgrade Characterization

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Geostatistics for Subgrade Characterization

Final Report

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SUMMARY

A detailed geostatistical analysis was carried out for subgrade modulus values inferred from falling weight deflectometer (FWD) measurements for five different roads around the State of Minnesota. Justified by the results of this analysis, subgrade modulus values for roads around the State of Minnesota can be effectively modelled as spatially correlated lognormal random variables.

The lognormal distribution model requires an uncommon set of equations to estimate the average, standard deviation, and percentiles. In addition, the lognormal distribution model suggests a modified calculation for the preliminary subgrade modulus design value. These new equations are presented in this report.

Based upon the geostatistical model, this report presents guidelines and nomographs for selecting the preliminary sample spacing for assessing the subgrade modulus. The maximum sample spacing to achieve a required precision is represented as a function of the average, standard deviation, and correlation length.

INTRODUCTION

This report presents some guidelines and procedures to help design the sampling plan for subgrade characterization on MnDOT projects. In addition, this report presents some modified calculations for analyzing and summarizing subgrade modulus data after they are collected, and for determining preliminary subgrade modulus design values.

These recommended guidelines and procedures are based upon two foundational observations from a recent detailed study of five road sections from different geologic domains around the state of Minnesota (Aboulkheir and Barnes, 1992).

- (1) On any project the subgrade modulus is variable, but this variability does not follow a classical symmetric bell-shaped curve. The distributions of subgrade modulus are not symmetric: in general, there is a longer tail toward the higher values.
- (2) On any project the subgrade modulus is variable, but this variability is not spatially random. Though there is a random component, on the average, two subgrade modulus measurements taken closer together are more similar than two measurements taken further apart.

The first observation motivates the application of a *lognormal distribution* model for subgrade modulus (see Section 1 for more details). The lognormal distribution model requires an uncommon set of equations to estimate the average, standard deviation, and percentiles. In addition, the lognormal distribution model suggests a modified calculation for the preliminary subgrade modulus design value. These new equations are presented in Sections 2, 3, and 4 of this report.

The second observation motivates the development of a *spatial correlation* model (e.g. *variogram*) for subgrade modulus. The critical parameter from this model is the *correlation length*. The correlation length is a quantitative means of expressing how quickly the subgrade modulus changes along the project. In some geologic settings the subgrade modulus varies rather slowly in space (a long correlation length), while in other geologic settings the subgrade modulus varies quite rapidly in space (a short correlation length). The estimation of the subgrade modulus correlation length for a project is discussed in Section 5 of this report.

Section 6 of this report presents guidelines and nomographs for selecting the preliminary sample spacing for assessing the subgrade modulus. The maximum sample spacing to achieve a required precision is represented as a function of the average, standard deviation, and correlation length. For example, to achieve a specified precision, as the standard deviation increases the necessary sample spacing decreases. On the other hand, as the correlation length increases so may the allowable sample spacing.

Appendix A presents a partial glossary of statistical and geostatistical terms and notation used in this report.

Each section of this report that presents a new tool (e.g. equation or nomograph) includes an example application of the tool. These examples are developed using the subgrade modulus data presented in Appendix B. These data were back-calculated from FWD measurements collected along TH52, between mile posts 123.5 and 126.1.

At the recommendation of the Mn/DOT Technical Panel, Appendix C is a reformatted version of Section 6 of this report, which is self-contained and suitable for physical separation from this larger report.

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1. HISTOGRAMS AND THE LOGNORMAL DISTRIBUTION

In most cases reported in the technical literature, and in all of the specific cases studied by Aboulkheir and Barnes (1992) within Minnesota, the distributions of subgrade modulus are not symmetric: there is a longer tail toward the higher values. An example is shown in Figure 1 below.

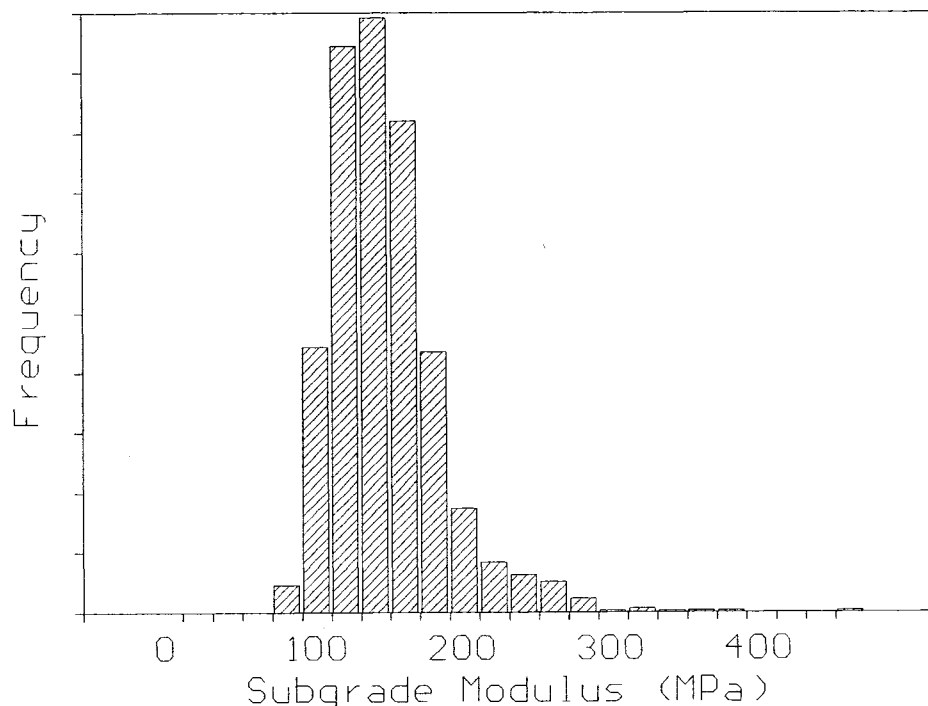


Figure 1. An example of a non-symmetric histogram of subgrade modulus. These 4092 data were back-calculated from FWD measurements collected in southern Minnesota, along TH30 between mile posts 57 and 97.

Such non-symmetric distributions can generally be well-modelled by a *two or three parameter lognormal distribution*. This means that the histogram of the logarithms of the subgrade modulus plots as a common bell-shaped curve (i.e. the Normal distribution). A detailed discussion of the Lognormal distribution for Civil Engineers can be found in Benjamin and Cornell (1970, p. 262-270); an abbreviated, but focused, presentation of the Lognormal distribution for subgrade modulus can be found in Aboulkheir and Barnes (1992). The practical consequences of the Lognormal distribution model for subgrade modulus are discussed in the next two sections of this report.

2. THE AVERAGE AND THE STANDARD DEVIATION

When it is necessary to estimate the average and the standard deviation of the subgrade modulus for a section of road from limited field measurements, the common formulae (e.g. those programmed into engineering calculators) are often inadequate due to the asymmetry of the distribution. A more appropriate procedure is based upon the three parameter lognormal distribution.

2.1. Manual Computations

Given "n" representative subgrade modulus data values, say $\{M_1, M_2, \dots, M_n\}$, the average and standard deviation can be estimated as follows.

Step 1) Using knowledge of the region, subjectively determine the likely minimum value for the subgrade modulus within the area of interest (δ). This value may be 0, but it must be less than the smallest measured value.

Step 2) Compute the intermediate variables, α and β , using Equations (1) and (2).

$$\alpha = \frac{1}{n} \sum_{i=1}^n \ln(M_i - \delta) \quad (1)$$

$$\beta^2 = \frac{1}{n} \sum_{i=1}^n [\ln(M_i - \delta) - \alpha]^2 \quad (2)$$

Note: the parameters " α " and " β " are merely intermediate place-holders to simplify the presentation of the necessary formulae. The " α " and " β " in Equations (1) and (2) are not associated with the " α " and " β " found in some introductory statistical textbooks under hypothesis testing and significance levels. While " α " and " β " can be correctly interpreted as the average and standard deviation of a transformed variable $y_i = \ln(M_i - \delta)$, such an interpretation does not add any significant insight or utility.

Step 3) Finally, compute the estimated average (μ^*) and standard deviation (σ^*) using Equations (3) and (4).

$$\mu^* = e^{\left(\alpha + \frac{\beta^2}{2}\right)} + \delta \quad (3)$$

$$\sigma^* = e^{\left(\alpha + \frac{\beta^2}{2}\right)} [e^{\beta^2} - 1]^{\frac{1}{2}} \quad (4)$$

2.2. Computer Computations

Using *Subgrade Geostatistics*, the microcomputer based program, optimal estimates for the average and standard deviation are automatically computed. In addition, this program computes the optimal shift parameter, along with confidence intervals on the mean and the shift parameter.

2.3. Example Computations

The three step procedure for manually estimating the average and standard deviation was applied to the example data set presented in Appendix B, with the following results:

Step 1) The smallest subgrade modulus value in the example data set is 19.2. Using external knowledge of the area from which this data was collected, the smallest likely value was selected to be: $\delta = 15$.

Step 2) Using Equations (1) and (2), and a hand-held engineering calculator, α and β were computed as

$$\begin{aligned}\alpha &= 2.723 \\ \beta &= 0.492\end{aligned}$$

Remember, " α " and " β " are merely intermediate place-holders and have no useful physical interpretation.

Step 3) Using Equations (3) and (4), the estimated average and standard deviation were computed as

$$\begin{aligned}\mu^* &= 32.2 \\ \sigma^* &= 9.0\end{aligned}$$

Using *Subgrade Geostatistics*, the estimated average (μ^*), standard deviation (σ^*), and shift parameter (δ), were computed for the example data set presented in Appendix B:

$$\begin{aligned}\mu^* &= 32.2 \\ \sigma^* &= 9.2 \\ \delta &= 15.9\end{aligned}$$

3. ESTIMATING PERCENTILES

Percentiles are often used as a basis for selecting preliminary and final design values for the subgrade modulus.

3.1. Manual Computations

3.1.1. Medium and Large Data Sets ($n > 50$)

When the representative data set is not small ($n > 50$), the best engineering estimate of an intermediate percentile of the subgrade modulus in the area of interest (e.g. the 16 percentile as a design value) is the associated percentile of the available data.

3.1.2. Small Data Sets ($n < 50$)

Though the direct computation of a percentile, as discussed in Section 4.1.1 above, is the preferred method, when the representative data set is small ($n < 100$), it is often necessary to incorporate additional information into the estimate of percentiles. Specifically, a distributional model is often necessary to augment the available measurements.

Given numerical values for the average (μ), the standard deviation (σ), and the shift parameter (δ), the "p" percentile of a three parameter lognormal distribution can be computed as follows.

Step 1) Compute the intermediate variables, α and β , using Equations (5) and (6).

$$\beta^2 = \ln \left[1 + \left(\frac{\sigma}{\mu - \delta} \right)^2 \right] \quad (5)$$

$$\alpha = \ln[\mu - \delta] - \frac{\beta^2}{2} \quad (6)$$

Note: as in the previous section, the parameters " α " and " β " are merely intermediate place-holders to simplify the presentation of the necessary formulae.

Step 2) Compute the "p" percentile (call it M_p) using Equation (7).

$$M_p = e^{(\alpha - Z_p \beta)} + \delta \quad (7)$$

where Z_p cuts off (1-p)percent of the lower tail of the standard normal distribution. Specific values of Z_p are given in Table 1 below.

Table 1. Z_p cuts off $(1-\nu)$ percent of the lower tail of the standard normal distribution.

P	Z_p
0.05	1.64
0.10	1.28
0.15	1.04
0.20	0.84
0.25	0.67
0.30	0.53
0.35	0.39
0.40	0.25

3.2. Computer Computations

Using *Subgrade Geostatistics*, the microcomputer based program, estimates for a set of percentiles are automatically computed. In addition, this program computes the associated confidence intervals on the estimates.

3.3. Example Computations

Starting with the average, standard deviation, and shift parameter developed in Section 3.3 for the example data set presented in Appendix B,

$$\begin{aligned}\mu^* &= 32.2 \\ \sigma^* &= 9.2 \\ \delta &= 15.9\end{aligned}$$

the two step procedure outlined in Section 3.1 was applied to estimate the 20 percentile with the following results:

Step 1) Using Equations (5) and (6), the intermediate variables α and β were computed as

$$\begin{aligned}\alpha &= 2.6529 \\ \beta &= 0.5258\end{aligned}$$

Step 2) From Table 1, the value for Z_p is

$$Z_{0.20} = 0.84$$

and the application of Equation (7) yielded

$$M_{0.20} = 25.0 .$$

As the example data set, presented in Appendix B, contains 74 data values, a direct estimation of the 20 percentile can be carried out with reasonable precision.

$$0.20 * 74 = 14.8$$

so, the 14th value in a sorted list of the data is a direct estimate of the 20 percentile:

$$M_{0.20} = 25.0 .$$

Thus, in this instance, both manual methods estimate identical values for the 20 percentile.

Using *Subgrade Geostatistics*, the estimated 20 percentile is also 25.0.

4. SELECTING PRELIMINARY DESIGN VALUES

The selection of an appropriate subgrade modulus value for preliminary design is a task made more difficult by two interrelated factors: uncertainty and variability. The uncertainty is a consequence of estimating the average subgrade modulus based upon a relatively small number of measurements. The variability is a consequence of geology: soils are not homogeneous, and the subgrade modulus will change, more or less, along any project. Given a preliminary design value for the subgrade modulus, some locations along a project will have a higher modulus, others a lower modulus.

4.1. Statistically-based Design Values

Fortunately, statistical modelling (e.g. the Lognormal distribution) offers a consistent, unified, objective, quantitative means of selecting a preliminary design value for the subgrade modulus, even in the face of uncertainty and variability. For example, the Minnesota Department of Transportation road design policy is typical of the Upper Midwest. Mn/DOT (1982, Section 7-5.03.02) states:

R-values are difficult to determine ... As a result the design R-value must necessarily represent the considered judgement of the Soils Engineer in the resolution of test data scatter. For preliminary design an R-value near the mean minus one standard deviation of the test values obtained during preliminary soil survey is generally selected.

This practice of using a value considerably below the mean is justified by the rule of engineering conservatism. This is reinforced by the observation that the life-time cost of somewhat under-estimating the subgrade modulus and over-designing the pavement section is less than equivalently over-estimating the subgrade modulus and under-designing the section.

The specific selection of "the mean minus one standard deviation" given above, is historically based on an assumed bell-shaped normal distribution. If the distribution of subgrade modulus were normal, and an equivalent design criterion were used, then the associated design value would be the 16th percentile.

In order to allow for the varying asymmetry of the observed subgrade modulus distributions around the state, yet maintain the same level of design conservatism, the preliminary subgrade modulus design value should be computed as follows.

$$M_{design} = e^{(\alpha - \beta)} + \delta \quad (8)$$

where α and β are computed from μ , σ , and δ by Equations (5) and (6), or computed from representative data using Equations (1) and (2). Equation (8) is nothing more than Equation (7) with $Z_p = 1$.

4.2. Example Computations

Starting with the average, standard deviation, and shift parameter developed in Section 3.3 for the example data set presented in Appendix B,

$$\begin{aligned}\mu^* &= 32.2 \\ \sigma^* &= 9.2 \\ \delta &= 15.9\end{aligned}$$

the intermediate variables, α and β , were computed using Equations (5) and (6) as

$$\begin{aligned}\alpha &= 2.6529 \\ \beta &= 0.5258\end{aligned}$$

Applying Equation (8) yielded

$$M_{\text{design}} = 24.3 .$$

5. ESTIMATING THE CORRELATION LENGTH

5.1. Manual Computations

In general, it is impractical to carry out the computations necessary to estimate the correlation length using only a hand-held engineering calculator. However, when there is little or no representative field data from the area of interest, there are two useful heuristics for roughly estimating the correlation length.

First, if there are sufficient data from a neighboring area, and the geologic conditions are comparable, then the correlation length from the neighboring area can be used as an initial estimate of the correlation length for the area of interest.

Second, when there are insufficient quantitative data from in and around the area of interest, then qualitative data can be used to form an initial estimate of the correlation length. Using visible characteristics (e.g. soil color and texture, presence of rock outcrops, vegetation, and moisture condition), partition the area of interest into qualitatively homogeneous zones. One third to one half of the average zone size can be used as an initial estimate of the correlation length.

5.2. Computer Computations

Using *Subgrade Geostatistics*, the microcomputer based program, estimates for the correlation length can be computed using the variogram modelling capabilities.

5.3. Example Computations

Using *Subgrade Geostatistics*, the estimated correlation length for the example data set, presented in Appendix B, was computed as

0.043 miles (exponential variogram model)

6. PRELIMINARY SAMPLING DESIGN

6.1. Introduction

The preliminary sample spacing is a function of four factors:

- sampling objective,
- desired precision,
- project size, and the
- geologic setting as characterized by the correlation length.

The first two factors require engineering decisions; the objective and the precision are not intrinsic variables, they must be selected based upon need, economics, and engineering judgement. The project size is the length of the project measured as a number of correlation lengths, where the correlation length is estimated from existing data or inferred from neighboring projects or project of a similar geologic character.

There are three sampling objectives considered in this document: (1) to estimate the average subgrade modulus value along the entire project; (2) to establish a preliminary design subgrade modulus value; and (3) to interpolate subgrade modulus values at various locations along the project. Each of these three objectives will be considered in turn below.

6.2. Relative Precision

Throughout this document "precision" refers to the relative precision of the estimated or inferred quantity. The relative precision is defined as to one standard deviation of the estimation error expressed as a percentage of the estimated quantity. For example, a relative precision of 0.10, when estimating the average subgrade modulus along the project, indicates that there is roughly a two-thirds chance that the true average modulus is within plus or minus 10% of the estimated average modulus.

In general, it is appropriate to use the common Normal Distribution ("bell-shaped curve") percentiles to develop approximate confidence intervals from the relative precision. Thus, there is a 68% chance that the true value is within plus or minus one standard deviations, and a 95% chance that the true value is within plus or minus two standard deviations.

6.3. Estimating the Project Average

Figure 2, titled "Project Average", can be used as a nomograph for selecting an appropriate minimum sample spacing to achieve a desired precision for the estimated average subgrade modulus over an entire project. For example, if the project size over the correlation length is eight (8), then twelve (12) samples would achieve a relative precision of approximately 5%.

This nomograph is based upon the following assumptions: (1) the project average is estimated by the arithmetic average of the available samples; (2) each sample is comprised of the average of three or more FWD drops; and (3) the samples are collected uniformly along the project.

An atlas of the State of Minnesota showing the approximate correlation length of roadway segments will be developed. Until this atlas is completed, previously collected FWD data can be processed through the

geostatistic's computer program (*Subgrade Gestatistics*) to determine the correlation length.

6.4. Estimating the Design Value

Figure 3, titled "Design Value", can be used as a nomograph for selecting an appropriate minimum sample spacing to achieve a desired precision for the estimated design subgrade modulus. For example, if the project size over the correlation length is eight (8), then twenty (20) samples would achieve a relative precision of approximately 5%.

This nomograph is based upon the following assumptions: (1) the design subgrade modulus value is the "mean minus one standard deviation" (e.g. Mn/DOT, 1982, Section 7-5.03.02), or equivalently, the 16 percentile of subgrade modulus values along the project; (2) each sample is comprised of the average of three or more FWD drops; and (3) the samples are collected uniformly along the project.

6.5. Local Interpolation

Figure 4, titled "Local Interpolation", can be used as a nomograph for selecting an appropriate minimum sample spacing to achieve a desired precision for the interpolation of the subgrade modulus between sampled locations. For example, if the sample spacing over the correlation length is one (1.0), then the relative precision of an interpolated value midway between two samples would be approximately 27%.

This nomograph is based upon the following assumptions: (1) the interpolated subgrade modulus value is computed using linear interpolation between the two neighboring samples; and (2) each sample is comprised of the average of three or more FWD drops.

6.6. Background Assumptions

There are three fundamental geostatistical modelling decisions that were made during the creation of the three nomographs. First, the spatial variation of the subgrade modulus is well modeled by an exponential semi-variogram. Second, the relative short range variability (relative nugget effect) is approximately 30%. Third, the population coefficient of variation for subgrade modulus is approximately 35%. These three modelling decisions are representative of the five Minnesota roads studied in Aboulkheir and Barnes (1992).

Figure 2. Project Average

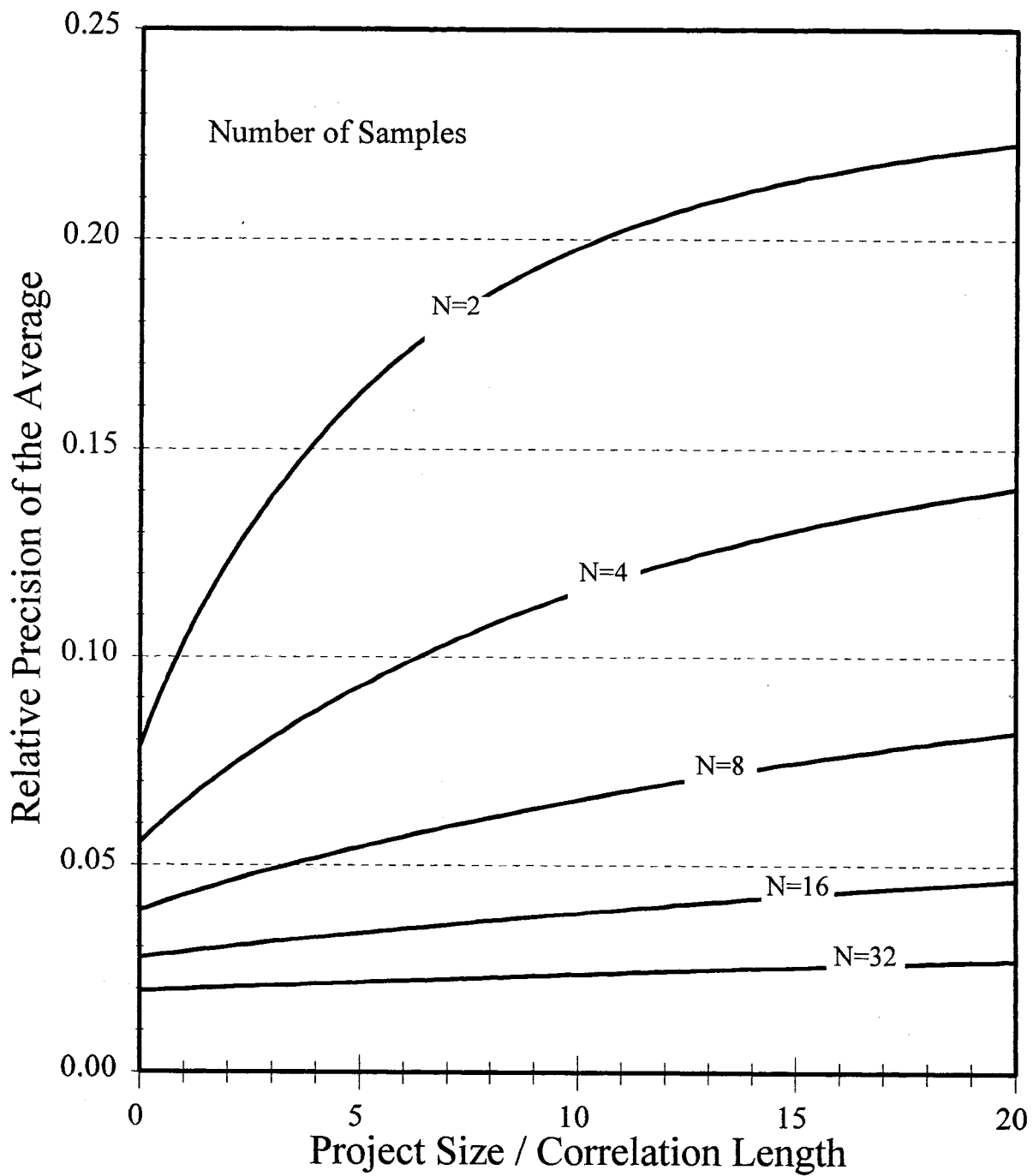


Figure 3. Design Value

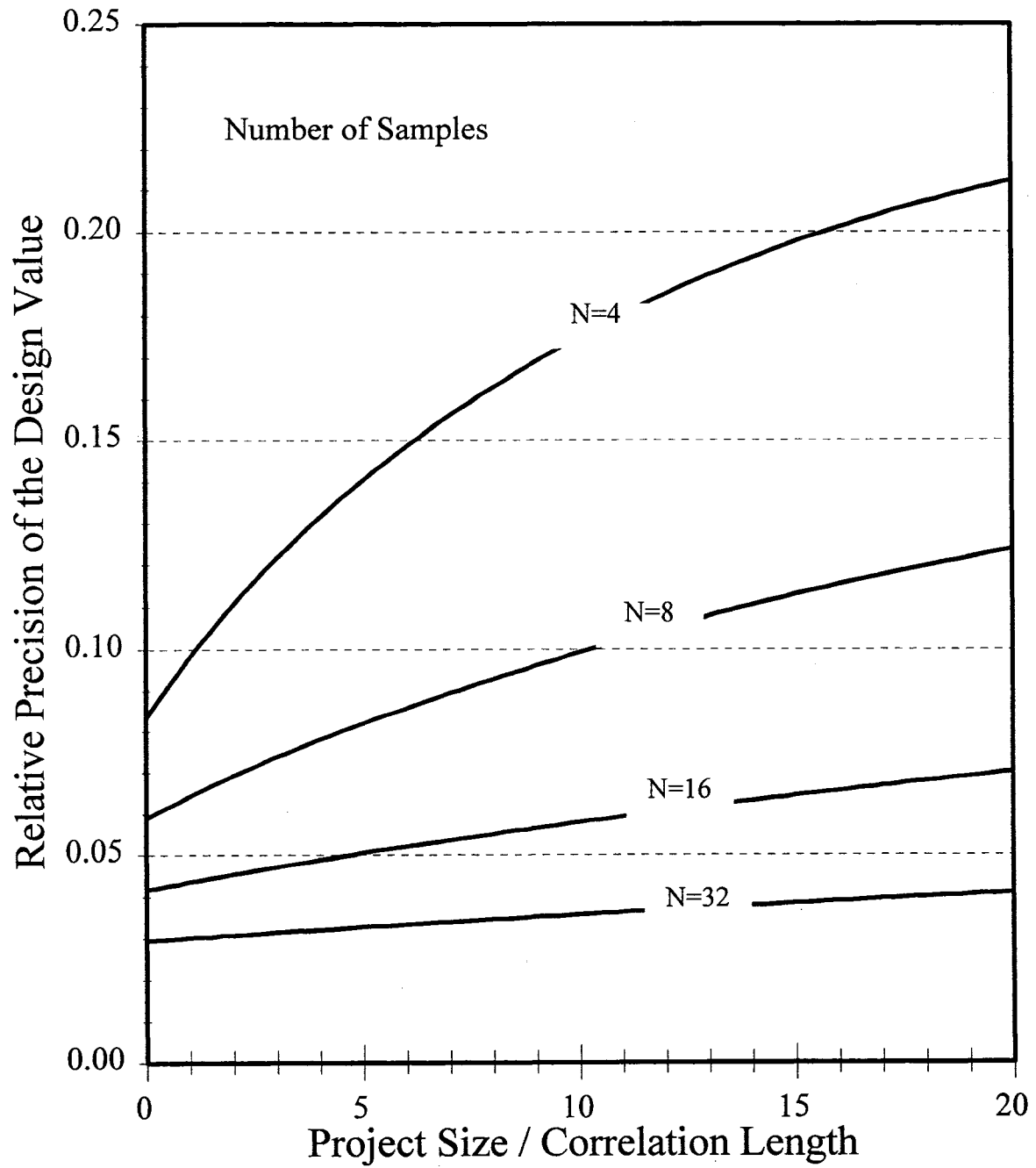
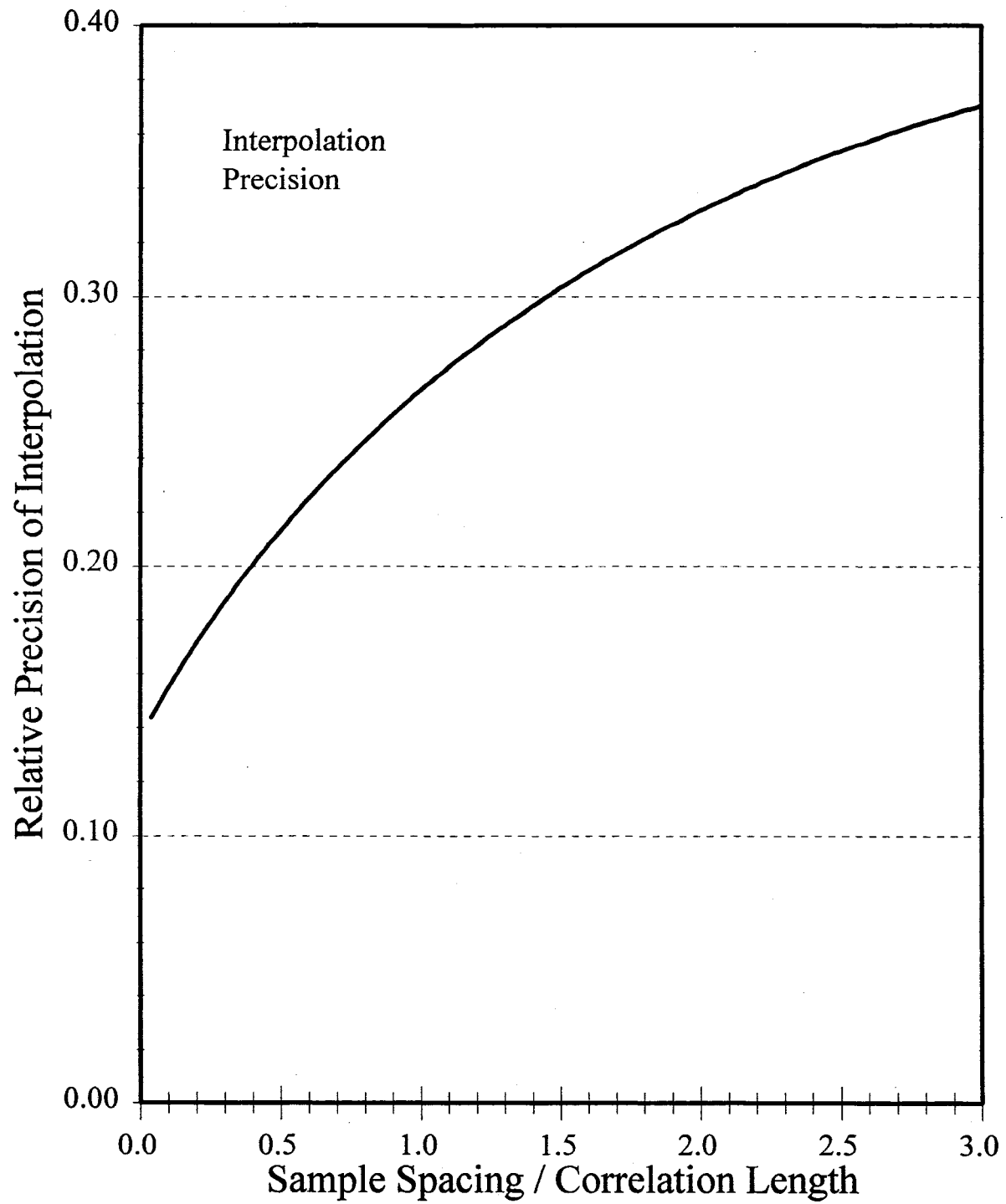


Figure 4. Local Interpolation



7. CITED REFERENCES

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Benjamin, J. R., and Cornell, C. A., 1970, **Probability, Statistics, and Decision for Civil Engineers**, McGraw-Hill Book Company, New York, 684 pp.

Mn/DOT, 1982, **Road Design Manual**, Part II (January 31, 1982), Minnesota Department of Transportation.

APPENDIX A - Glossary of Terms and Notation

APPENDIX A - Glossary of Terms and Notation

- α . The parameter " α " is merely intermediate place-holders to simplify the presentation of the necessary formulae. The " α " in Equation (1) is not associated with the " α " found in some introductory statistical textbooks under hypothesis testing and significance levels. While " α " can be correctly interpreted as the average of a transformed variable $Y_i = \ln(M_i - \delta)$, such an interpretation does not add any significant insight or utility.
- β . The parameter " β " is merely intermediate place-holders to simplify the presentation of the necessary formulae. The " β " in Equation (2) is not associated with the " β " found in some introductory statistical textbooks under hypothesis testing and significance levels. While " β " can be correctly interpreted as the standard deviation of a transformed variable $Y_i = \ln(M_i - \delta)$, such an interpretation does not add any significant insight or utility.
- δ . The parameter " δ " represents the shift parameter in the *three parameter lognormal distribution* model for the subgrade modulus values.
- μ . The parameter " μ " represents the true underlying average of the distribution of subgrade modulus values. A superscript "*" indicates that the value is an estimate of the true average.
- σ . The parameter " σ " represents the true underlying standard deviation of the distribution of subgrade modulus values. A superscript "*" indicates that the value is an estimate of the true standard deviation.
- Co. The parameter "Co" represents the short-range variability. Specifically, it is the repeatability variance of subgrade modulus measurements. In the field of geostatistics, Co is often called the "nugget effect".
- coefficient of variation.** The coefficient of variation is a dimensionless measure of variability. It is defined as the standard deviation (σ) divided by the average (μ).
- correlation length.** The correlation length is a quantitative means of expressing how quickly the subgrade modulus changes along the project. In some geologic settings the subgrade modulus varies rather slowly in space (a long correlation length), while in other geologic settings the subgrade modulus varies quite rapidly in space (a short correlation length).
- exponential variogram.** The exponential variogram (or exponential semivariogram) is a specific common semivariogram model. This model was found to be a reasonable descriptor for subgrade modulus in Minnesota.
- lognormal distribution.** The lognormal distribution is the most common probability distribution model for geologic variables. The lognormal distribution is a non-symmetric probability distribution. There are two different forms of the lognormal distribution: a *two parameter lognormal distribution* and a *three parameter lognormal distribution*.

M_i . The subscripted variable " M_i " is used to represent the i 'th subgrade modulus measurement.

relative precision. The relative precision is a dimensionless measure of precision. It is usually expressed as a percent error. For example, an estimate of 50 plus or minus 10 would have a relative precision of 20%.

scaled relative precision. In this report, a scaled relative precision is a relative precision divided by the population coefficient of variation. This scaling by the coefficient of variation allows the creation of a single plot or nomograph encompassing a wide variety of situations.

sill. See the entry for "variogram sill".

spatial correlation. The spatial correlation is a statistically formal means of capturing the following observation: on the average, two subgrade modulus measurements taken closer together are more similar than two measurements taken further apart. The spatial correlation quantifies the relationship between the separation distance and the degree of similarity.

standard normal distribution. The standard normal distribution is a specific normal distribution (the common bell-shaped curve) with average equal to 0 and standard deviation equal to 1.

Subgrade Geostatistics. Within this report, *Subgrade Geostatistics* is a large computer program for carrying out statistical analysis of FWD and subgrade modulus data. This program was written for Mn/DOT by Dr. R.J. Barnes of the University of Minnesota.

three parameter lognormal distribution. The three parameter lognormal distribution is a particular form of the general *lognormal distribution* with three defining parameters: the *average* (μ), the *standard deviation* (σ), and a shift parameter (δ). There is a simple relationship between the three parameter lognormal distribution and the classic normal distribution (the bell-shaped curve): if X is distributed according to the three parameter lognormal distribution with average (μ), standard deviation (σ), and shift parameter (δ), then $Y = \ln(X-\delta)$ is distributed normally with average (α) and standard deviation (β).

two parameter lognormal distribution. The two parameter lognormal distribution is a particular form of the general *lognormal distribution* with two defining parameters: the *average* " μ " and the *standard deviation* " σ ". The two parameter lognormal distribution is the same as the three parameter lognormal distribution with the shift parameter (δ) set equal to zero.

variogram. The variogram (or semi-variogram) is a common representation of spatial correlation. The variogram is commonly used in geostatistics. The variogram is commonly described with three parameters: *Co*, *Sill*, and *Correlation Length*.

variogram sill. The variogram sill is a measure of the population variability. Under fairly general modelling conditions, the variogram sill is equal to the population variance (standard deviation squared).

APPENDIX B - Example Data Set

APPENDIX B - Example Data Set

TH52 - Mile Post 123.5 to 126.1
(Mile Post, Subgrade Modulus)

123.511	28.5	123.985	26.9	124.360	48.7
123.531	35.0	124.000	32.9	124.379	37.8
123.550	33.6	124.002	25.9	124.398	45.4
123.570	23.0	124.019	26.7	124.417	24.6
123.587	27.6	124.021	41.3	124.436	24.7
123.606	46.2	124.038	27.4	124.456	24.0
123.625	39.7	124.039	26.8	124.495	32.0
123.643	22.5	124.057	33.1	124.600	48.8
123.665	43.4	124.058	27.7	124.700	24.8
123.683	45.9	124.076	39.8	124.800	48.6
123.701	29.9	124.077	36.0	124.900	33.0
123.720	34.9	124.095	31.9	125.000	25.1
123.739	26.0	124.117	45.8	125.100	27.7
123.758	37.6	124.134	43.5	125.200	29.6
123.777	39.7	124.153	49.1	125.300	22.3
123.796	24.7	124.171	63.9	125.400	27.5
123.814	20.1	124.189	25.3	125.500	19.2
123.833	26.1	124.211	29.5	125.600	26.5
123.852	25.0	124.227	39.2	125.697	28.6
123.871	24.4	124.247	33.4	125.797	27.0
123.890	25.3	124.265	31.9	125.900	31.4
123.909	28.7	124.284	26.7	126.000	32.7
123.928	26.0	124.303	25.5	126.097	35.7
123.947	22.5	124.322	33.7	126.141	47.7
123.966	23.2	124.341	25.2		

APPENDIX C - Preliminary Sampling Design

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1. INTRODUCTION. The preliminary sample spacing is a function of four factors:

- sampling objective,
- desired precision,
- project size, and the
- geologic setting as characterized by the correlation length.

The first two factors require engineering decisions; the objective and the precision are not intrinsic variables, they must be selected based upon need, economics, and engineering judgement. The project size is the length of the project measured as a number of correlation lengths, where the correlation length is estimated from existing data or inferred from neighboring projects or project of a similar geologic character.

There are three sampling objectives considered in this document: (1) to estimate the average subgrade modulus value along the entire project; (2) to establish a preliminary design subgrade modulus value; and (3) to interpolate subgrade modulus values at various locations along the project. Each of these three objectives will be considered in turn below.

2. RELATIVE PRECISION. Throughout this document "precision" refers to the relative precision of the estimated or inferred quantity. The relative precision is defined as to one standard deviation of the estimation error expressed as a percentage of the estimated quantity. For example, a relative precision of 0.10, when estimating the average subgrade modulus along the project, indicates that there is roughly a two-thirds chance that the true average modulus is within plus or minus 10% of the estimated average modulus.

In general, it is appropriate to use the common Normal Distribution ("bell-shaped curve") percentiles to develop approximate confidence intervals from the relative precision. Thus, there is a 68% chance that the true value is within plus or minus one standard deviations, and a 95% chance that the true value is within plus or minus two standard deviations.

3. ESTIMATING THE PROJECT AVERAGE. The figure titled "Project Average" can be used as a nomograph for selecting an appropriate minimum sample spacing to achieve a desired precision for the estimated average subgrade modulus over an entire project. For example, if the project size over the correlation length is eight (8), then twelve (12) samples would achieve a relative precision of approximately 5%.

This nomograph is based upon the following assumptions: (1) the project average is estimated by the arithmetic average of the available samples; (2) each sample is comprised of the average of three or more FWD drops; and (3) the samples are collected uniformly along the project.

An atlas of the State of Minnesota showing the approximate correlation length of roadway segments will be developed. Until this atlas is completed, previously collected FWD data can be processed through the geostatistic's computer program (*Subgrade Gestatistics*) to determine the correlation length.

4. ESTIMATING THE DESIGN VALUE. The figure titled "Design Value" can be used as a nomograph for selecting an appropriate minimum sample spacing to achieve a desired precision for the estimated design subgrade modulus. For example, if the project size over the correlation length is eight (8), then twenty (20) samples would achieve a relative precision of approximately 5%.

This nomograph is based upon the following assumptions: (1) the design subgrade modulus value is the "mean minus one standard deviation" (e.g. Mn/DOT, 1982, Section 7-5.03.02), or equivalently, the 16 percentile of subgrade modulus values along the project; (2) each sample is comprised of the average of three or more FWD drops; and (3) the samples are collected uniformly along the project.

5. LOCAL INTERPOLATION. The figure titled "Local Interpolation" can be used as a nomograph for selecting an appropriate minimum sample spacing to achieve a desired precision for the interpolation of the subgrade modulus between sampled locations. For example, if the sample spacing over the correlation length is one (1.0), then the relative precision of an interpolated value midway between two samples would be approximately 27%.

This nomograph is based upon the following assumptions: (1) the interpolated subgrade modulus value is computed using linear interpolation between the two neighboring samples; and (2) each sample is comprised of the average of three or more FWD drops.

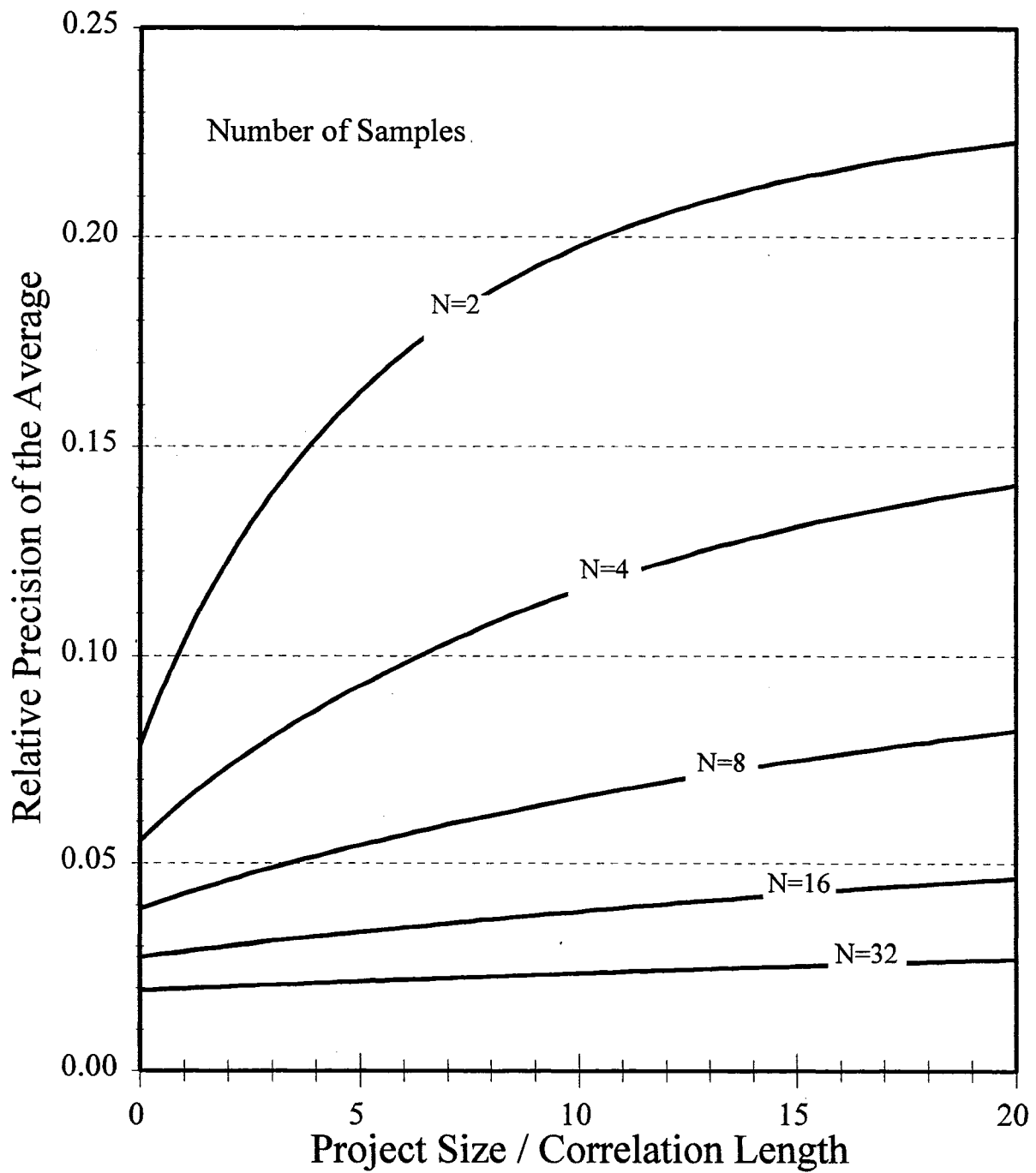
6. BACKGROUND ASSUMPTIONS. There are three fundamental geostatistical modelling decisions that were made during the creation of the three nomographs. First, the spatial variation of the subgrade modulus is well modeled by an exponential semi-variogram. Second, the relative short range variability (relative nugget effect) is approximately 30%. Third, the population coefficient of variation for subgrade modulus is approximately 35%. These three modelling decisions are representative of the five Minnesota roads studied in Aboulkheir and Barnes (1992).

7. CITED REFERENCES.

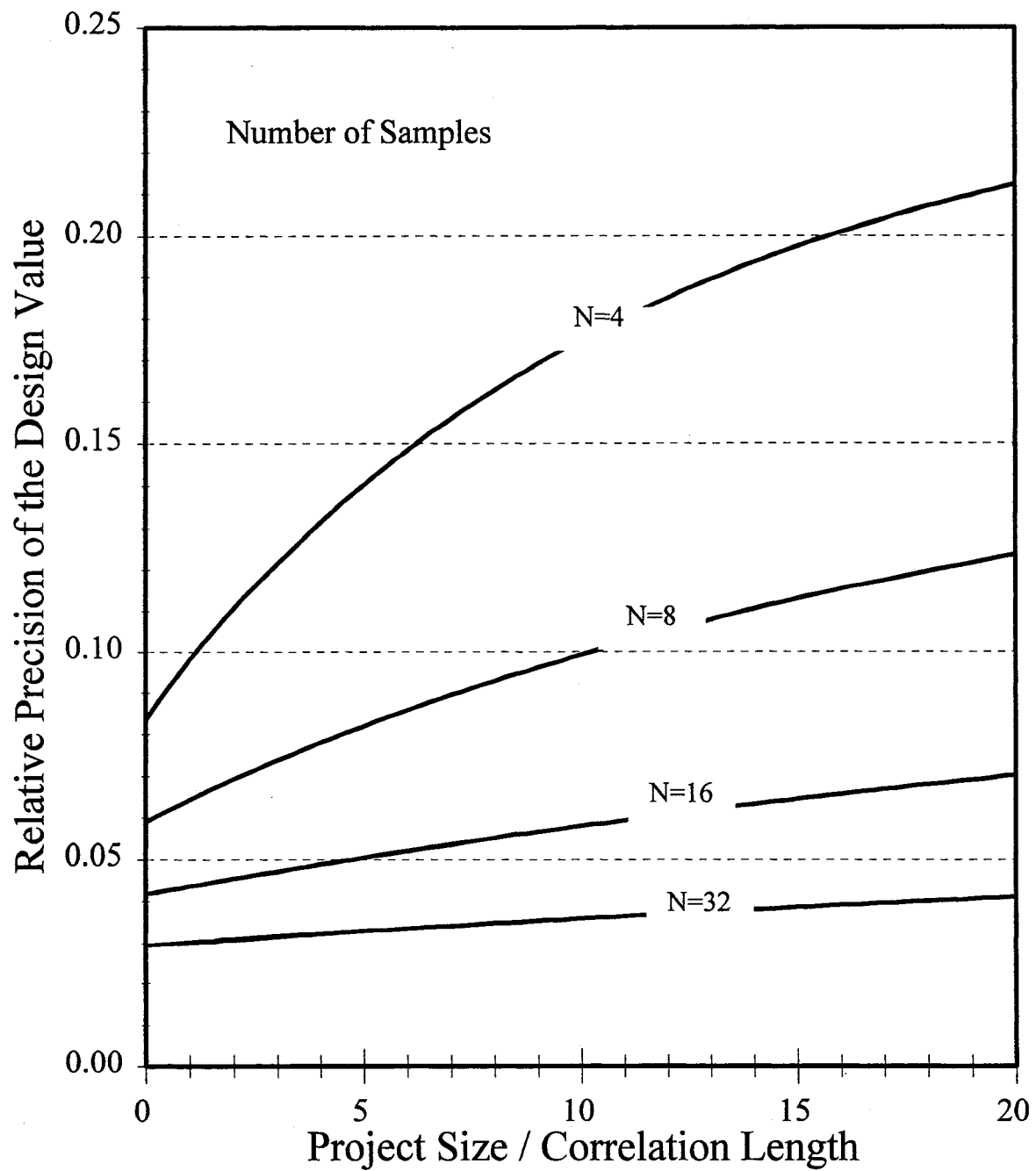
Aboulkheir, L. and Barnes, R., 1992, **Geostatistics for Pavement Design on Variable Soils**, Department of Civil and Mineral Engineering, University of Minnesota, Minneapolis, 233 pp.

Mn/DOT, 1982, **Road Design Manual**, Part II (January 31, 1982), Minnesota Department of Transportation.

Project Average



Design Value



Local Interpolation

